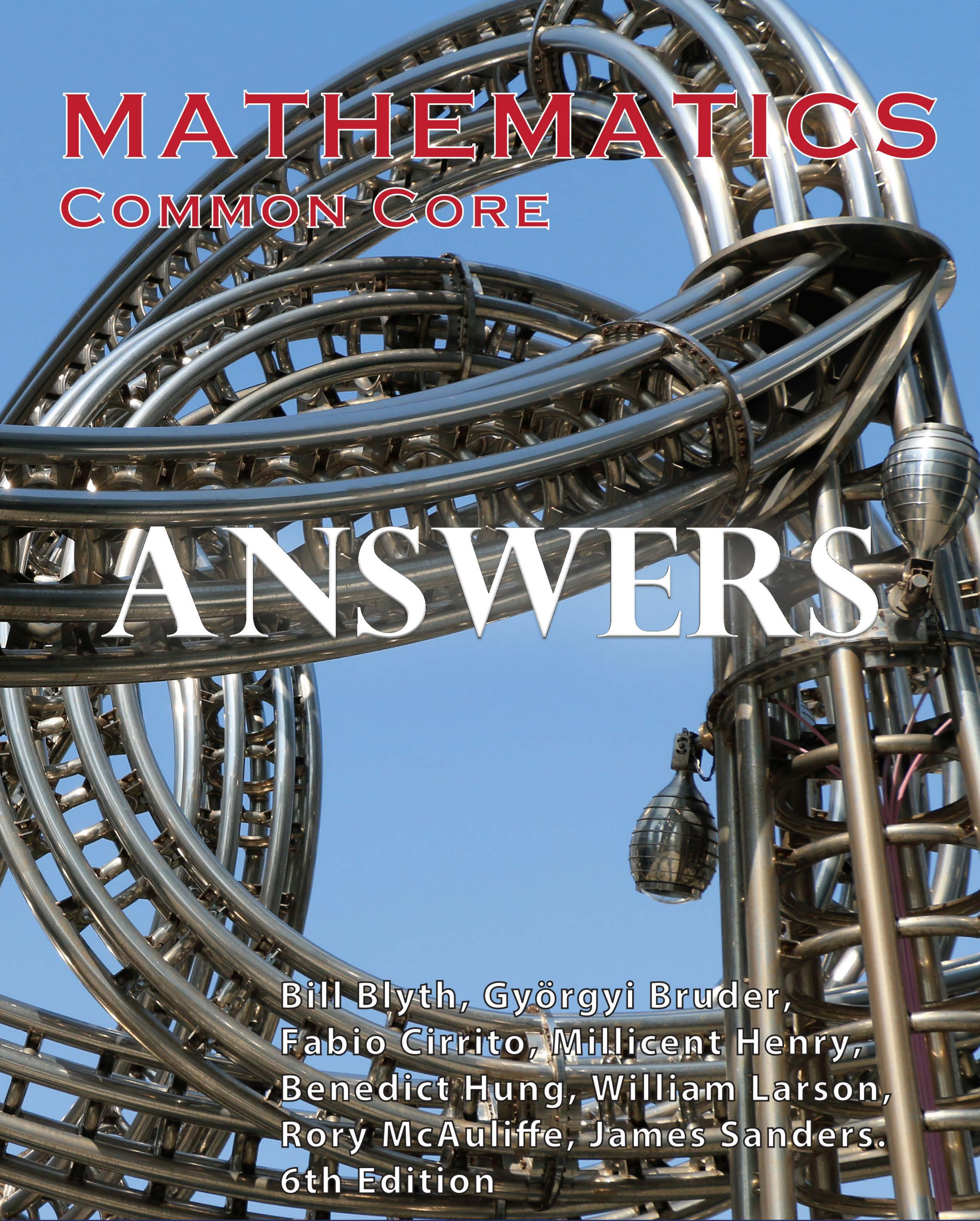

MATHEMATICS

Common Core

ANSWERS

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6th Edition

For use with the I.B. Diploma Programme

Exercise A.1.1

1.	a	5.771×10^{21}	b	3.635×10^{8}	c	2.003×10^{6}
	d	1.855×10^{8}	e	2.539×10^{19}	f	1.613×10^{10}
	g	5.081×10^{13}	h	4.711×10^{9}	i	3.992×10^{7}
	j	4.616×10^{4}	k	2.714×10^{0}	l	1.334×10^{33}
	m	8.429×10^{29}	n	6.704×10^{9}		
2.	a	2.386×10^{-18}	b	9.630×10^{-5}	c	7.567×10^{-13}
	d	8.235×10^{-16}	e	1.589×10^{-7}	f	2.898×10^{1}
	g	7.695×10^{-6}	h	2.899×10^{-11}	i	4.379×10^{-34}
	j	1.076×10^{-8}	k	1.154×10^{-1}	l	7.498×10^{-34}
	m	9.734×10^{-20}	n	3.634×10^{-12}		

3. 1.25×10^{8} mm^3 or 1.25×10^{-1} m^3

4. Area $\approx 5.101 \times 10^{14}$ m^2, Volume $\approx 1.083 \times 10^{21}$ m^3

5. The distance is given to 2 significant figures (SF). Even though the speed is given to a much higher level of accuracy, the answer should not be given to more than 2SF. Time is about 500 sec or about 8 minutes.

6. The least accurate item of data is 3 significant figures and so the answer should not exceed this ... 8.98×10^{56} atoms.

7. 15

8. 5×10^{14} s^{-1}

9. Number of Primes < 10^{12} $\dfrac{10^{12}}{\log_e 10^{12}} \approx 3.619\times10^{10}$. Number of Primes < 10^{13} $\dfrac{10^{13}}{\log_e 10^{13}} \approx 3.341\times10^{11}$

   Number of Primes between 10^{12} and 10^{13} = 2.979×10^{11}. This is about 3%.

10. An n sided polygon has $\dfrac{n(n-3)}{2}$ sides. 5×10^{11} diagonals.

Exercise A.2.1

1. i **b** 4 **c** $t_n = 4n - 2$
   ii **b** –3 **c** $t_n = -3n + 23$
   iii **b** –5 **c** $t_n = -5n + 6$
   iv **b** 0.5 **c** $t_n = 0.5n$
   v **b** 2 **c** $t_n = y + 2n - 1$
   vi **b** –2 **c** $t_n = x - 2n + 4$

2 –28

3 9,17

4 –43

5 7

6 7

7 –5

8 0

9 **a** 41 **b** 31st

10 2, $\sqrt{3}$

11 **a i** 2 **ii** –3 **b i** 4 **ii** 11

12 $x - 8y$

13 $t_n = 5 + \frac{10}{3}(n-1)$

14 **a** –1 **b** 0

Exercise A.2.2

1 **a** 145 **b** 300 **c** –170

2 **a** –18 **b** 690 **c** 70.4

3 **a** –105 **b** 507 **c** 224

4 **a** 126 **b** 3900 **c** 14th week

5 855

6 **a** 420 **b** –210

7 $a = 9, b = 7$

Exercise A.2.3

1 123

2 –3, –0.5, 2, 4.5, 7, 9.5, 12

3 3.25

4 $a = 3 \quad d = -0.05$

5 10 000

6 330

7 –20

8 328

9 $725, 37 weeks

10 **a** $55 **b** 2750

11 **a i** 8 m **ii** 40 m **b** 84 m
   c Dist = $2n^2 - 2n = 2n(n-1)$
   d 8 **e** 26 players, 1300 m

12 **a** 5050 **b** 10200 **c** 4233

13 **a** 145 **b** 390 **c** –1845

14 **b** $3n - 2$

Exercise A.2.4

1 **a** $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$

b $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$

c $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$

d $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$

e $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$

f $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$

2 **a** ± 12 **b** $\frac{\pm\sqrt{5}}{2}$

3 **a** ± 96 **b** 15th

4 **a** $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ **b** $\frac{15625}{3888} \cong 4.02$ **c** $n = 5$ 4 times

5 $-2, \frac{4}{3}$

6 **a i** \$4096 **ii** \$2097.15 **b** 6.2 yrs

7 $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \cong 471.16$

8 2.5,5,10 or 10,5,2.5

9 53 757

10 108 952

11 **a** \$56 156 **b** \$299 284

Exercise A.2.5

1 **a** 3 **b** $\frac{1}{3}$ **c** -1 **d** $-\frac{1}{3}$ **e** 1.25
f $-\frac{2}{3}$

2 **a** 216513 **b** 1.6384×10^{-10} **c** $\frac{256}{729}$
d $\frac{729}{2401}$ **e** $-\frac{81}{1024}$

3 **a** 11; 354 292 **b** 7; 473 **c** 8; 90.90909
d 8; 172.778 **e** 5; 2.256 **f** 13; 111.1111111111

4 **a** $\frac{127}{128}$ **b** $\frac{63}{8}$ **c** $\frac{130}{81}$
d 60 **e** $\frac{63}{64}$

5 4; 118 096

6 \$2109.50

7 9.28 cm

8 **a** $V_n = V_0 \times 0.7^n$ **b** 7

9 54

10 53.5 gms; 50 weeks.

11 7

12 9

13 $-0.5, -0.7797$

14 $r = 5,\ 1.8 \times 10^{10}$

15 $8407.35

16 1.8×10^{19} or about 200 billion tonnes.

Exercise A.2.6

1 Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047.

2 18

3 12

4 7, 12

5 8 weeks Ken $220 & Bo-Youn $255)

6 **a** week 8 **b** week 12

7 **a** 1.618

b 121 379 [~121400, depends on rounding errors]

Exercise A.2.7

1 **a** $\frac{81}{2}$ **b** $\frac{10}{13}$ **c** 5000 **d** $\frac{30}{11}$

2 $23\frac{23}{99}$

3 6667 fish. [NB: $t_{43} < 1$. If we use $n = 43$ then ans is 6660 fish]; 20 000 fish.

Overfishing means that fewer fish are caught in the long run.

4 27

5 48,12,3 or 16,12,9

6 **a** $\frac{11}{30}$ **b** $\frac{37}{99}$ **c** $\frac{191}{90}$

7 128 cm

8 $\frac{121}{9}$

9 $2 + \frac{4}{3}\sqrt{3}$

10 $\frac{1-(-t)^n}{1+t}$ $\frac{1}{1+t}$

11 $\frac{1-(-t^2)^n}{1+t^2}$ $\frac{1}{1+t^2}$

Exercise A.2.8

1 3, −0.2

2 $\frac{2560}{93}$

3 $\frac{10}{3}$

4 **a** $\frac{43}{18}$ **b** $\frac{458}{99}$ **c** $\frac{413}{990}$

5 9900

6 3275

7 3

8 $t_n = 6n - 14$

9 6

10 $-\frac{1}{6}$

11 **a** 12 **b** 26

12 9, 12

13	±2					
14	(5, 5, 5), (5, –10, 20)					
15	**a**	2, 7	**b**	2, 5, 8	**c**	$3n - 1$
16	**a**	5	**b**	2 m		

Exercise A.2.9

1	$2773.08
2	$4377.63
3	$1781.94
4	$12 216
5	$35 816.95
6	$40 349.37
7	$64 006.80
8	$276 971.93, $281 325.41
9	$63 762.25
10	$98.62, $9467.14, interest $4467.14. Flat interest = $6000
11	$134.41, $3790.44, 0.602% /month (or 7.22% p.a.)
12	–½, 3 The sequence ½, ½, ½,.. is arithmetic.
13	15
14	Proof
15	$m = 19$, $n = 34$

Exercise A.3.1

1.	2^9	2.	5^{10}	3.	2^5
4.	7^3	5.	3^9	6.	11^{12}
7.	3^{10}	8.	3^{13}	9.	9^{13}

Exercise A.3.2

1. a $2^2.3^2$ b $5^2.7^2$ c $2^3.3.5$

   d $2^3.3^2.7^2$ e $2.3.13$ f $2.3.5^3.7$

   g $2.3.5.7$ h $2.3.5^5$

2. a $\frac{b^3}{a^4}$ b $\frac{1}{2}x^5y^3$ c $\frac{8}{x^3}$

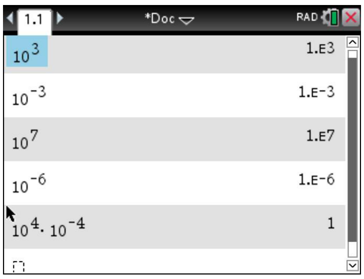
   d $\frac{1}{4x^5y^2}$ e $\frac{bc}{8a^2}$ f $\frac{1}{2q^2}$

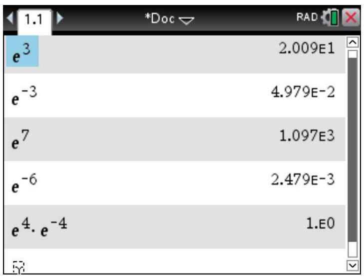
3. a $\frac{y}{2x}$ b $a+b+c$ c qr d $\frac{x}{y}$

4. ~393 cm

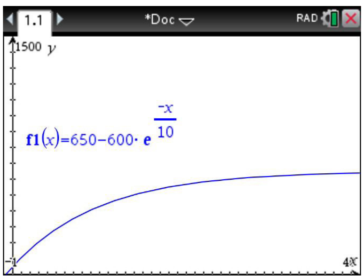
5. About 0.15 of a cubic centimetre.

Exercise A.3.3

1. 

2. 

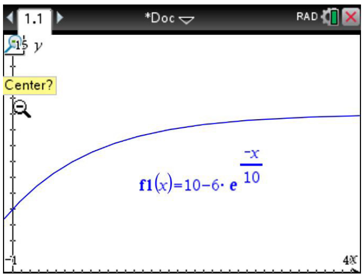
Exercise A.3.4

1. 

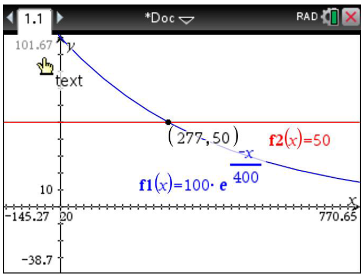
   This is a typical population growth pattern. The slowdown is caused by the declining food supply (as food is consumed).

2. Every 12 hours.

3.



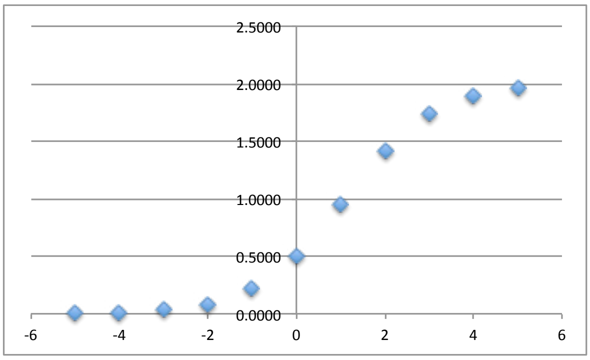
4.



Catalyst needs to be replaced after approx. 277 hours.

5.

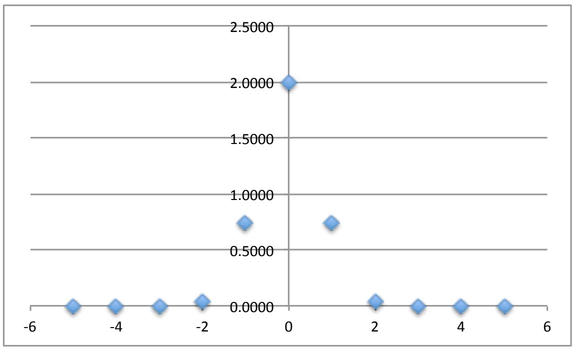
x	Expression
-5	0.0045
-4	0.0121
-3	0.0326
-2	0.0863
-1	0.2185
0	0.5000
1	0.9507
2	1.4225
3	1.7401
4	1.8958
5	1.9604



This sort of model applies to a range of situations where growth is initially rapid but which is limited by factors such as limited food supplies. It is also common in economics.

6.

x	Expression
-5	0.0000
-4	0.0000
-3	0.0002
-2	0.0366
-1	0.7358
0	2.0000
1	0.7358
2	0.0366
3	0.0002
4	0.0000
5	0.0000



This is the basis for the normal probability distribution. Deviations from the mean are random.

Exercise A.3.7

1. $262 \times 2^{-3} = 32.75$ and $262 \times 2^{6} = 16\,768$ which is about 10 octaves.

2. a 14 b 37.6 c ~50%

3. 10 dB

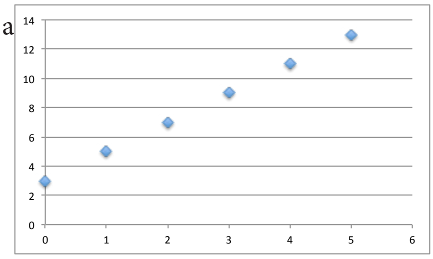
4. 100

5. a 10 mg/L b 8.4 mg/L c 12 hours

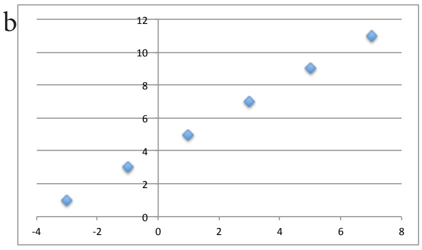
6. a 10 b 18 c ~3.95 years

Exercise B.2.1

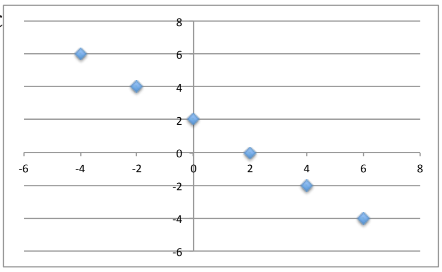
1. a



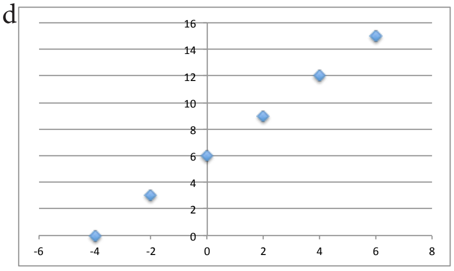
b



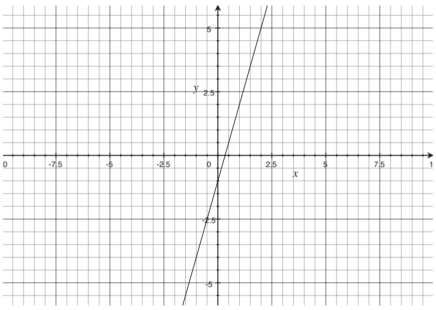
c



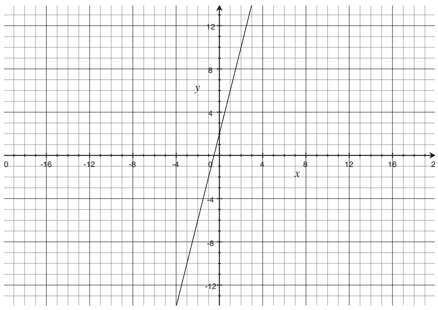
d



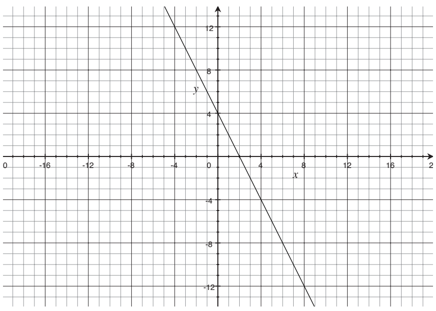
2. a



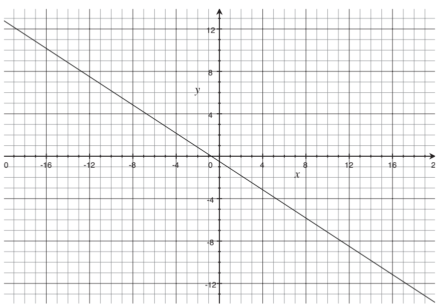
b



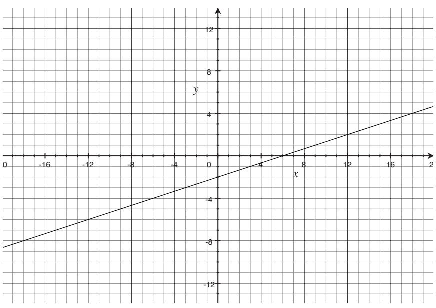
c



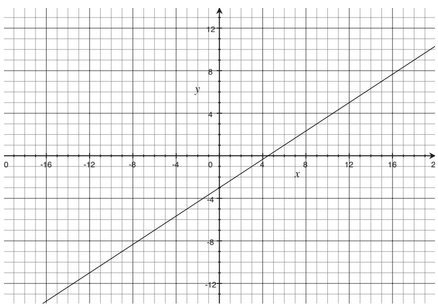
d



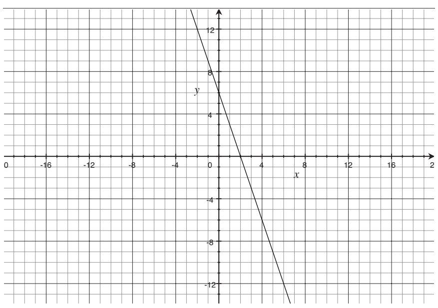
3. a



b



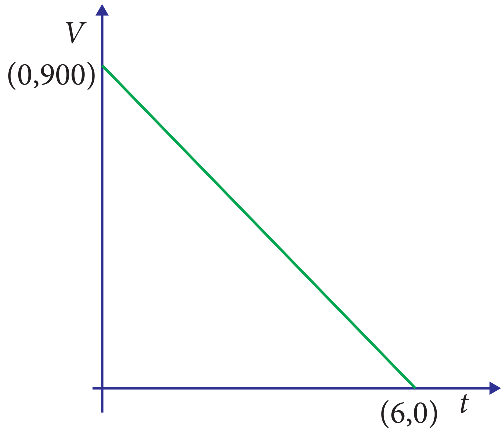
c



4. a

t (years)	0	1	2	3	4	5	6
V ($)	900	750	600	450	300	150	0

b

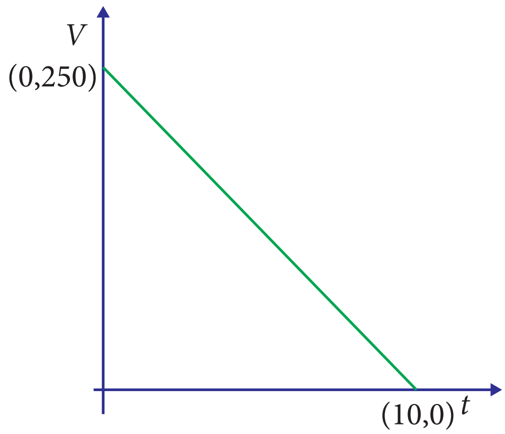


c Value of computer ≥ $0

5. a

t (hours)	0	1	2	3	4	5	...	9	10
V (Litres)	250	225	200	175	150	125		25	0

b

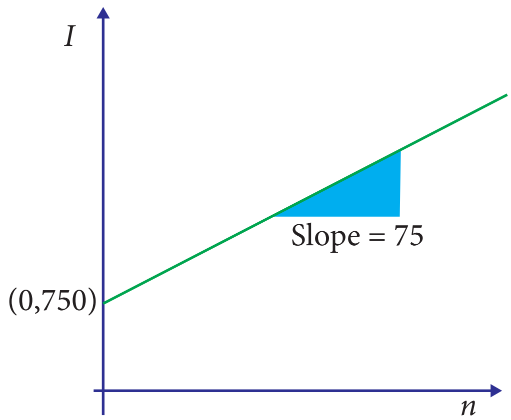


c $V \geq 0, 0 \leq t \leq 10$

6. a

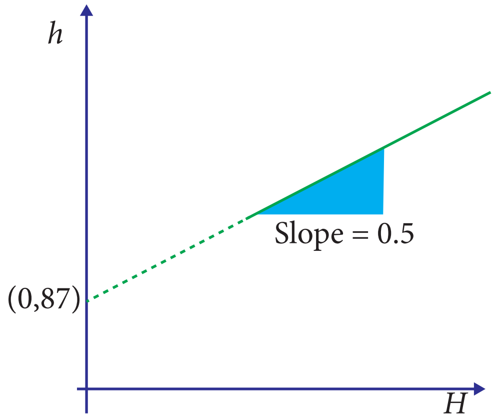
Sales (n)	0	1	2	3	...
Income ($$I$)	750	225	300	375	...

b,c



7. a An appropriate range for the father's height would be:

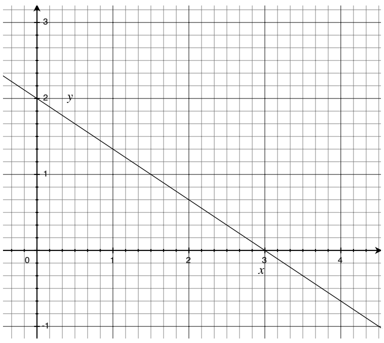
H	140	150	160	170	180	190
h	157	162	167	172	177	182



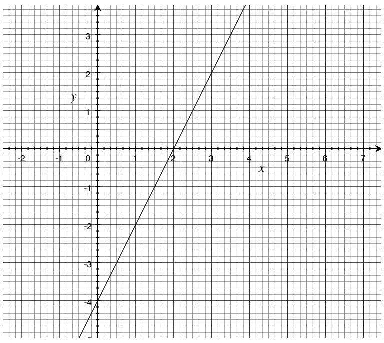
b 180.5 cm

Exercise B.1.2

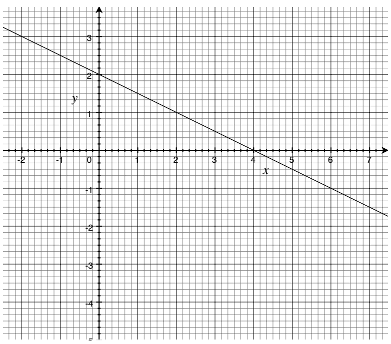
1. a



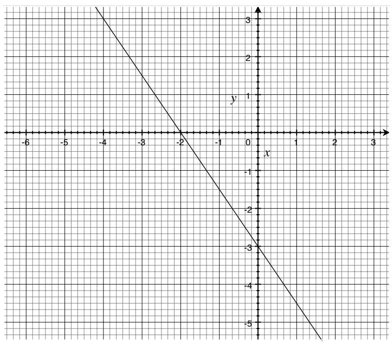
b



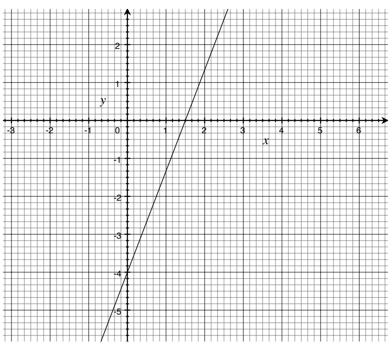
c



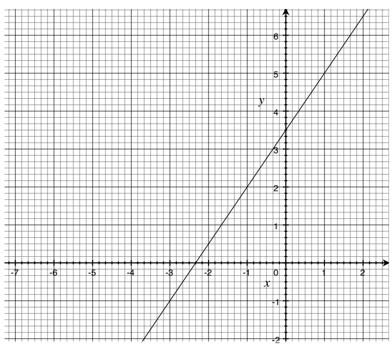
d

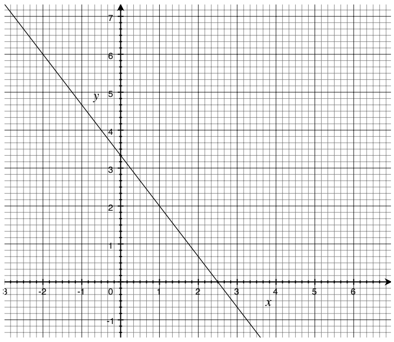


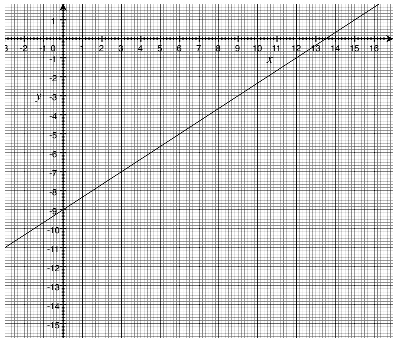
e



f



g 

h 

2. a $y = 2x + 5$ b $y = -x + 3$ c $y = -2x + 8$

d $y = -0.5x + 3$ e $3y = 2x$

a has a gradient of 2 and passes through the point (0, 5)

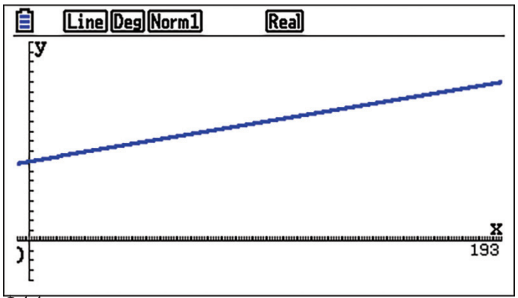
b has a gradient of –1 and passes through the point (0, 3)

c has a gradient of –2 and passes through the point (4, 0)

d has a gradient of –0.5 and passes through the point (4, 1)

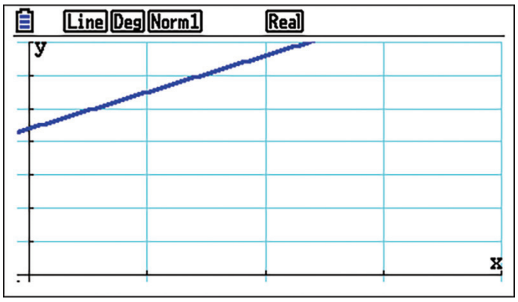
e has a gradient of $^2/_3$ and passes through the point (6, 4).

3. a $C = 4 + 0.02n$, $n = 0, 1, 2, 3...$

b 

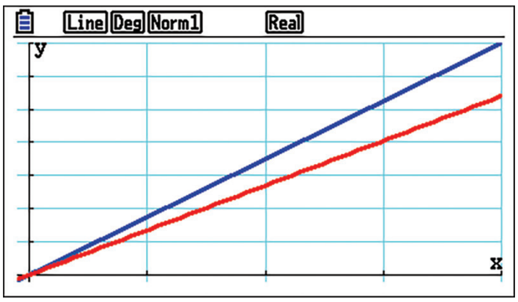
c $44

4. a $B = 44 + 0.22n$, $n = 0, 1, 2, ...$

b 

c $80.96 d 192

5. a i $P = 0.35k$ ii $G = 0.27k$

b 

c 12500 km

6. $ly = x + 1$

7. $x + y = 3$

8. $x + 2y = 15$

9. $y = x - 2$

Exercise B.2.1

2 Find the range for each of the following.

i $y = \sqrt{x}, x \geq 0$

j $y = \sqrt{x}, 1 \leq x \leq 25$

k $y = \frac{4}{x+1}, x > 0$

l $\{(x,y):y^2 = x, x \geq 1\}$

4 Determine the implied domain for each of the following relations.

h $y = \sqrt{x+a}, a > 0$

i $y = \frac{a}{\sqrt{x-a}}, a > 0$

j $x^2 - y^2 = a^2$

k $y^2 - x^2 = a^2$

5 Find the range of the following relations.

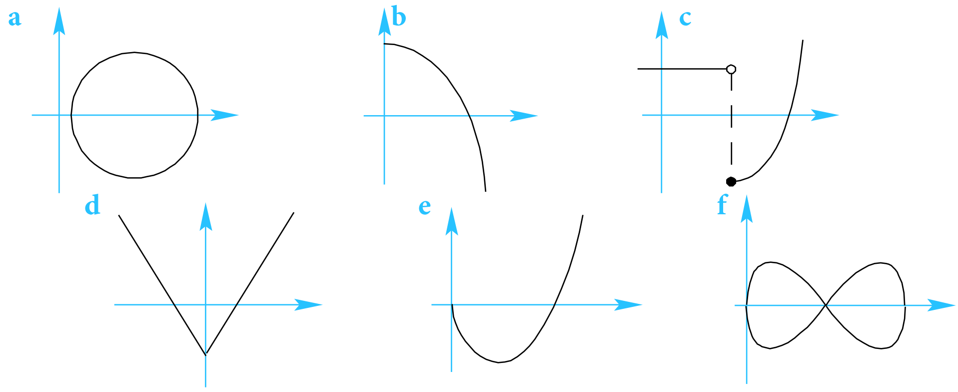
f $y = a - \frac{a}{x^2}, a > 0$

g $y = 2\sqrt{x-a} - a, a > 0$

h $y = \frac{2a}{\sqrt{a^2 - x}}, a < 0$

Exercise B.2.2

6 Which of the following relations are also functions?



7 Use both visual and algebraic tests to show that the following relations are also functions:

a $x \mapsto x^3 + 2, x \in]0,5[$ b $x \mapsto \sqrt{x} + 1, x \in [0, 9[$

c $\{(x,y): y^3 = x + 1, x \in \mathbb{R}\}$ d $\{(x,y): y = x^2 + 1, x \in \mathbb{R}\}$

8 Use an algebraic method to decide which of the following relations are also functions:

a $f: x \mapsto \frac{1}{x}, x \in \mathbb{R}\backslash\{0\}$ b $\{(x,y): y^2 - x = 9, \ x \geq -9\}$

c $\{(x,y): y^2 - x^2 = 9, \ x \geq -9\}$ d $f(x) = \frac{1}{x^2} + 1, x \neq 0$

e $f(x) = 4 - 2x^2, x \in \mathbb{R}$ f $f: x \mapsto \frac{4}{x+1}, x \in \mathbb{R}\backslash\{-1\}$

9 Sketch the graph of $f: \mapsto \frac{x^2}{x^2+2}, x \in \mathbb{R}$ and use it to:

a show that f is a function b determine its range.

10 A function is defined by $f: x \mapsto \frac{x+10}{x-8}, x \neq 8$ and $x \geq 0$.

a Determine the range of f.

b Find the value of a such that $f(a) = a$.

11 Consider the functions $h(x) = \frac{1}{2}(2^x + 2^{-x})$ and $k(x) = \frac{1}{2}(2^x - 2^{-x})$.

a Show that $2[h(x)]^2 = h(2x) + 1$.

b If $[h(x)]^2 - [k(x)]^2 = a$, find the constant a.

12 Which of the following functions are identical? Explain.

a $f(x) = \frac{x}{x^2}$ and $h(x) = \frac{1}{x}$. b $f(x) = \frac{x^2}{x}$ and $h(x) = x$.

c $f(x) = x$ and $h(x) = \sqrt{x^2}$ d $f(x) = x$ and $h(x) = (\sqrt{x})^2$.

13 Find the largest possible subset X of $\mathbb{R}$, so that the following relations are one-to-one increasing functions:

a $f: X \to \mathbb{R}$, where $f(x) = x^2 + 6x + 10$ **b** $f: X \to \mathbb{R}$, where $f(x) = \sqrt{9 - x^2}$

c $f: X \to \mathbb{R}$, where $f(x) = \sqrt{x^2 - 9}$ **d** $f: X \to \mathbb{R}$, where $f(x) = \frac{1}{3x - x^2}$, $x \neq 0, 3$

14 An isosceles triangle ABC has two side lengths measuring 4 cm and a variable altitude. Let the altitude be denoted by x cm.

a Find, in terms of *x*, a relation for:

i its perimeter, $p(x)$ cm and specify its implied domain.

ii its area, $A(x)$ cm^2 and specify its implied domain.

b Sketch the graph of:

i $p(x)$ and determine its range.

ii $A(x)$ and determine its range.

Exercise B.2.1

1 a dom = {2, 3, −2}, ran = {4, −9, 9}
  b dom = {1, 2, 3, 5, 7, 9}, ran = {2, 3, 4, 6, 8, 10}
  c dom = {0, 1}, ran = {1, 2}

2 a]1, ∞[b [0, ∞[c]9, ∞[
  d]−∞, 1] e [−3, 3] f] −∞, ∞[
  g]−1, 0] h [0, 4] i [0, ∞[
  j [1, 5] k]0, 4[l]−∞, −1] ∪ [1, ∞[

3 a $r = [-1, \infty[$, $d = [0, 2[$ b $r = \{y : y \geq 0\} \backslash \{4\}$, $d = \mathbb{R}$
  c $r = [0, \infty[\backslash \{3\}$, $d = [-4, \infty[\backslash \{0\}$ d $r = [-2, 0[$, $d = [-1, 2[$
  e $r =]-\infty, \infty[$ $d =]-\infty, -3] \cup [3, \infty[$ f $r = [-4,4]$, $d = [0,8]$

4 a $\mathbb{R} \backslash \{-2\}$ b $]-\infty, 9[$ c $[-4,4]$
  d $]-\infty, -2] \cup [2, \infty[$ e $\mathbb{R} \backslash \{0\}$ f $\mathbb{R}$
  g $\mathbb{R} \backslash \{-1\}$ h $[-a, \infty[$ i $[0, \infty[\backslash \{a^2\}$
  j $]-\infty, -a] \cup [a, \infty[$ k $\mathbb{R}$ l $\mathbb{R} \backslash \{-a^{-1}\}$

5 a $]-\infty, -a[$ b $]0, ab]$ c $]-\infty, \frac{1}{4}a^3]$ d $[\frac{1}{4}a^3, \infty[$
  e $\mathbb{R} \backslash \{a\}$ f $]-\infty, a[$ g $[-a, \infty[$ h $]-\infty, 0[$

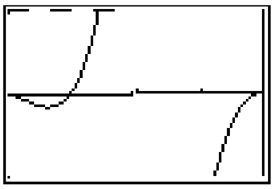
Exercise B.2.2

1 a 3, 5 b i $2(x+a) + 3$ ii 2a c 3

2 a $0, \frac{10}{11}$ b $-\frac{5}{4}$ c $\left[0, \frac{10}{11}\right]$

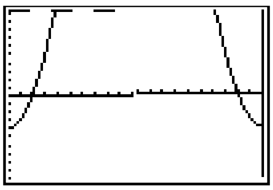
3 a $-\frac{1}{2}x^2 - x + \frac{3}{2}$, $-\frac{1}{2}x^2 + x + \frac{3}{2}$ b $\pm\sqrt{2}$ c no solution

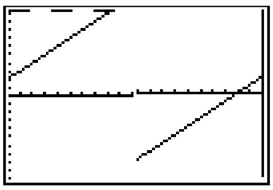
4 a x = 0, 1 **b**



Window [−2,2], [−1,1]
Range: [−12, 4]

5 ai **ii**



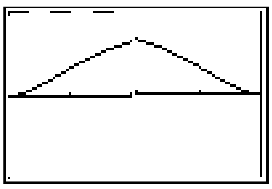


b i $\{2\sqrt{2}, -2\sqrt{2}\}$ ii {3, −2}

6 b, c, d, e

8 a, d, e, f

9 a Window [−2,2], [−1,1] **b** [0, 1[



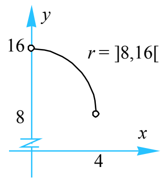
10 a $\{y : y > 1\} \cup \{y : y \leq -1.25\}$
  b 10

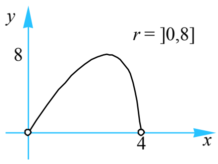
11 b 1

12 a only – it is the only one with identical rules and domains

13 a $[-3,\infty[$ b $[-3,0]$ c $[3,\infty[$
d $[1.5,3[\ \cup\ ]3,\infty[$

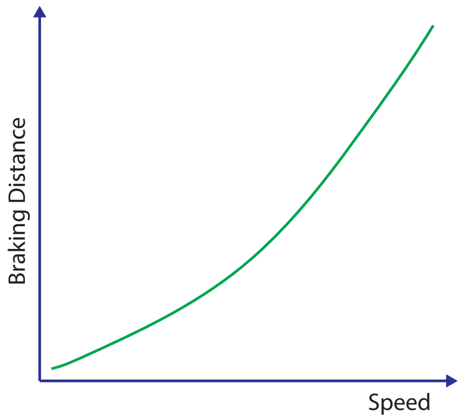
14 a i $p(x) = 8+2\sqrt{16-x^2}, 0<x<4$ ii $A(x) = x\sqrt{16-x^2}, 0<x<4$

b i 

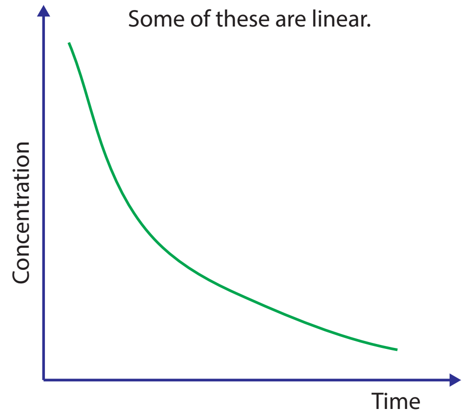
ii 

Exercise B.2.3

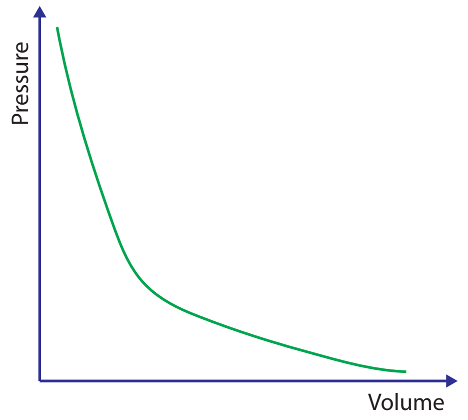
1.



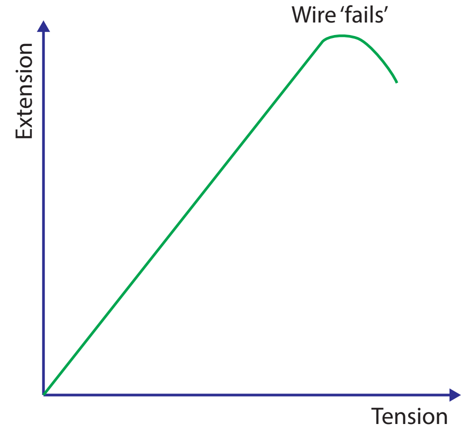
2.



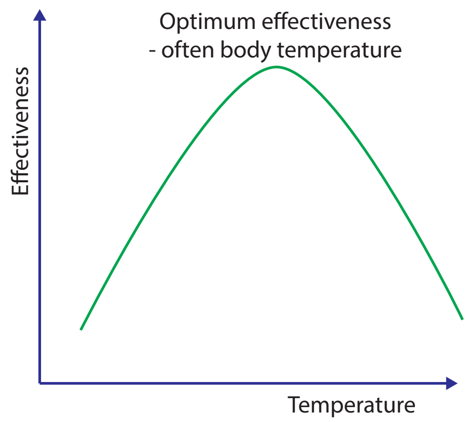
3.



4.

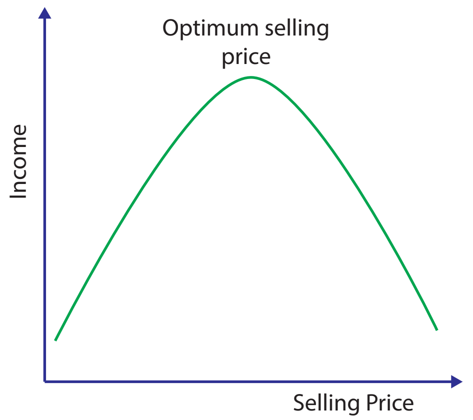


5.

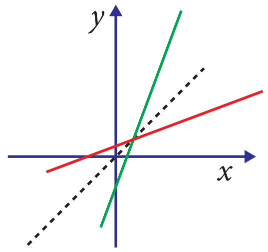


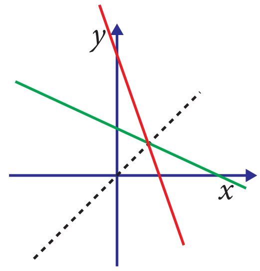
6.

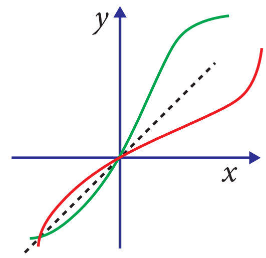
7.

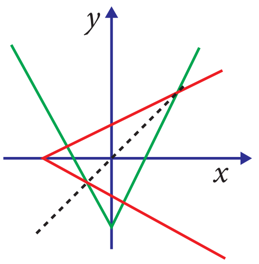


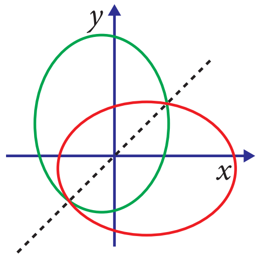
Exercise B.2.4

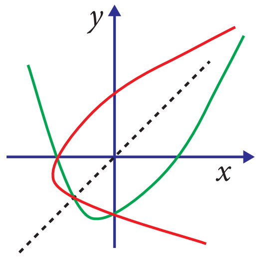
1. 

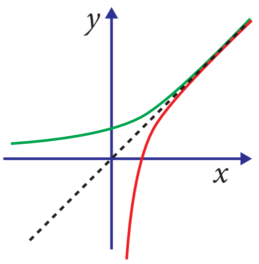
2. 

3. 

4. 

5. 

6. 

7. 

The inverse functions are: 1, 2, 3, 7.

Exercise B.3.1

1. a $h(x) = -1.6\times10^{-3}x^2 + 0.8x, 0 \le x \le 500$ b 32.76m

2. a 2.51×10^{-13} moles per litre b in terms of H-ion conc. $\frac{10^{-6.5}}{10^{-12.6}} \approx 1260000$

3. About 32 times.

4. 10^8 32 times.

5. $\sqrt{215}$ or approx 14.6 kph.

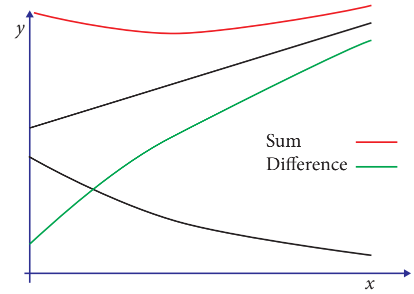
6. $\frac{5+\sqrt{73}}{2}$ hrs

7. i ~17.2m ii 12m

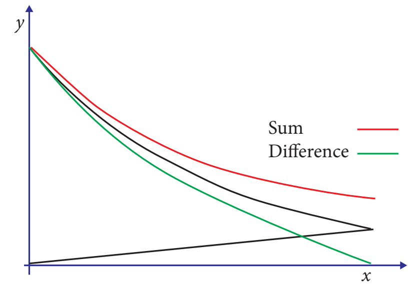
8. a ~332 000 b New payment ~8 900, new total interest 268 000 saving 64 000

Exercise B.3.2

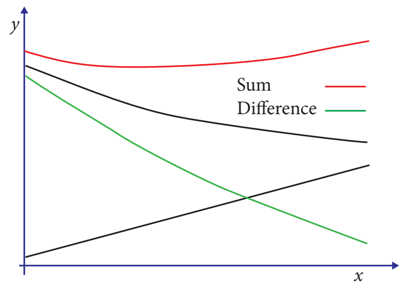
a



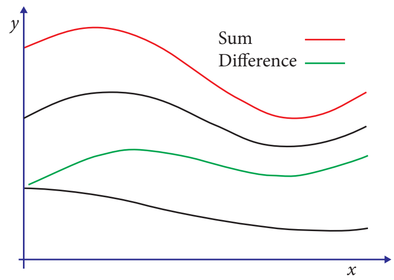
b



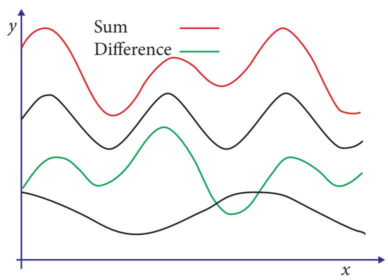
c



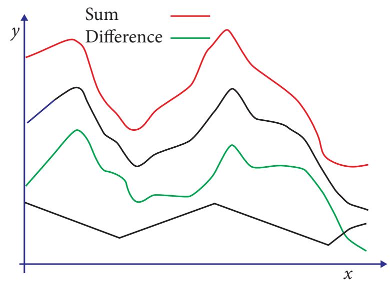
d



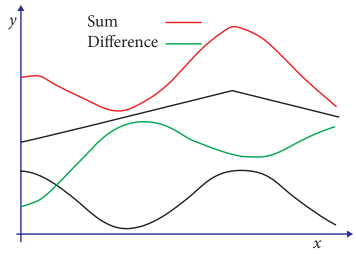
e



f

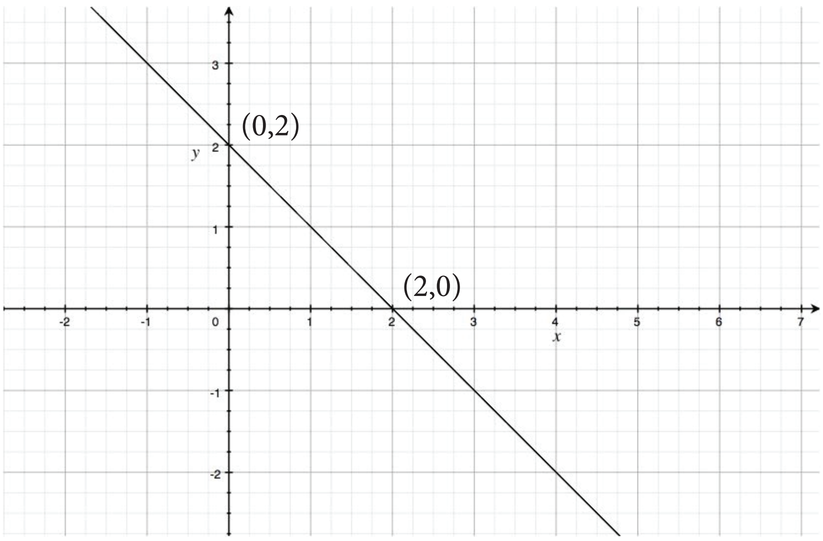


g

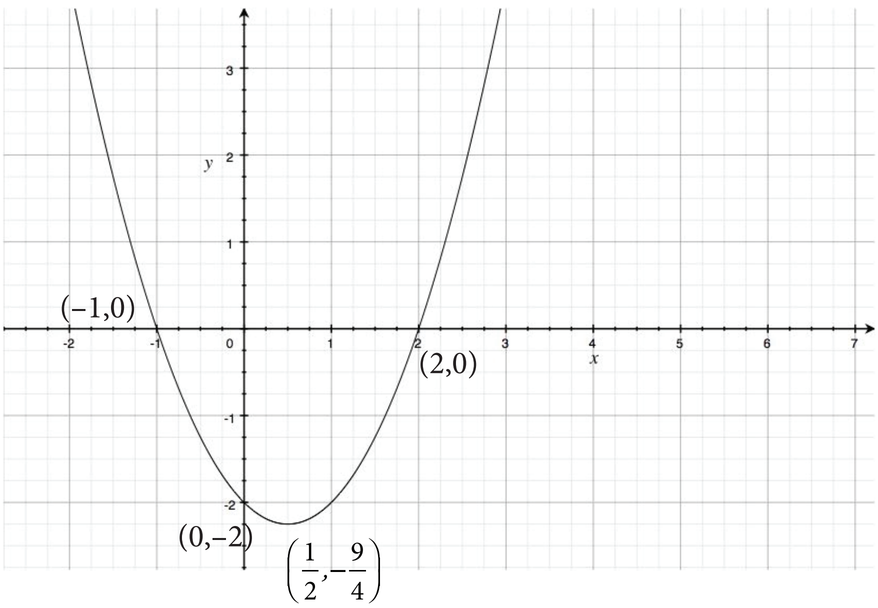


Exercise B.3.3

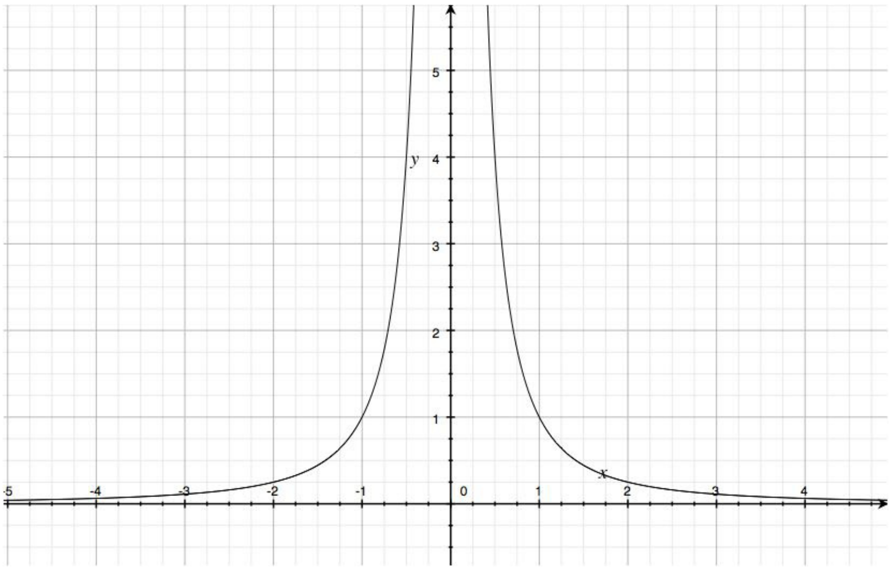
1.



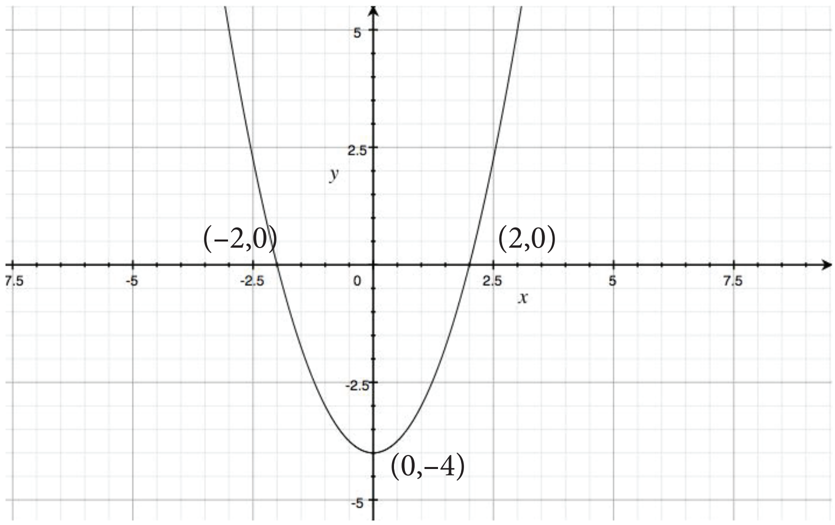
2.



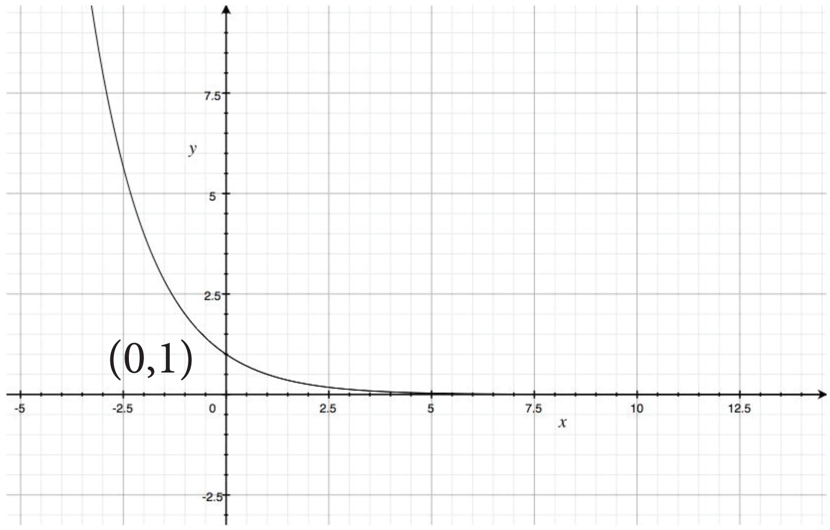
3.


y
x
5
4
3
2
1
-5
-4
-3
-2
-1
0
1
2
3
4

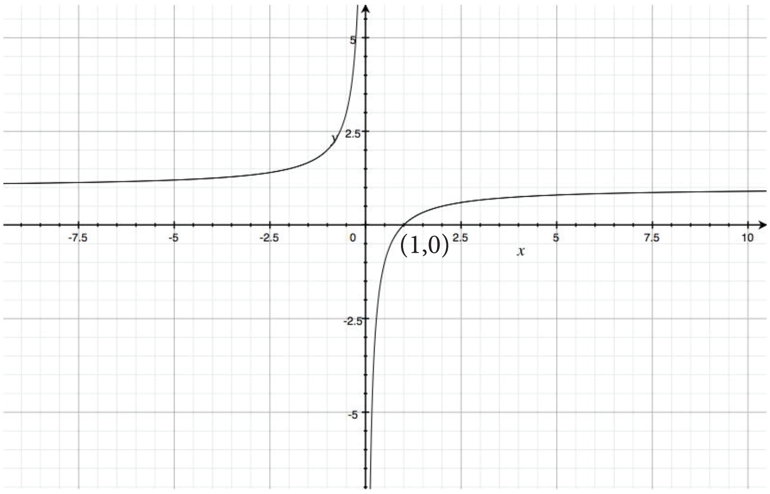
4.


y
x
5
2.5
-2.5
-5
7.5
-5
-2.5
0
2.5
5
7.5
(−2,0)
(2,0)
(0,−4)

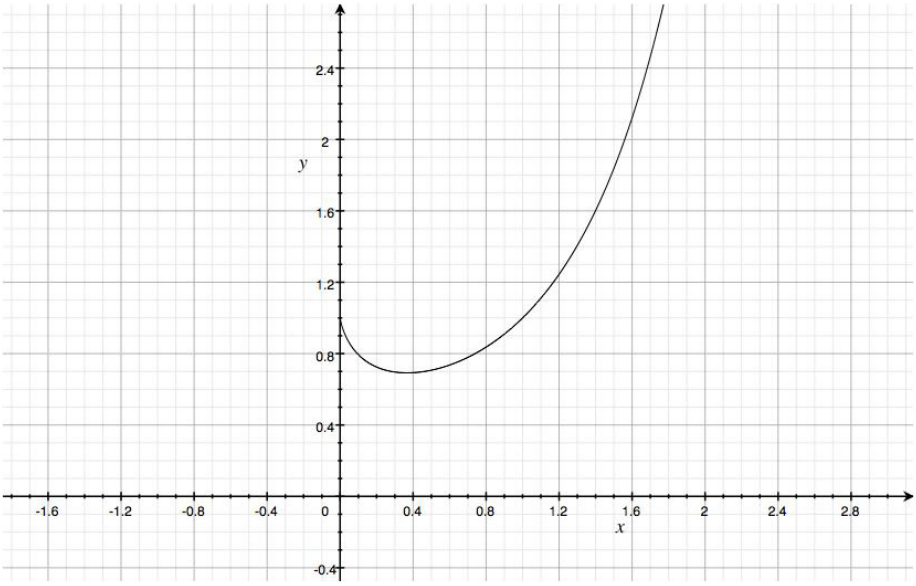
5.


y
x
7.5
5
2.5
-2.5
-5
-2.5
0
2.5
5
7.5
10
12.5
(0,1)

6.

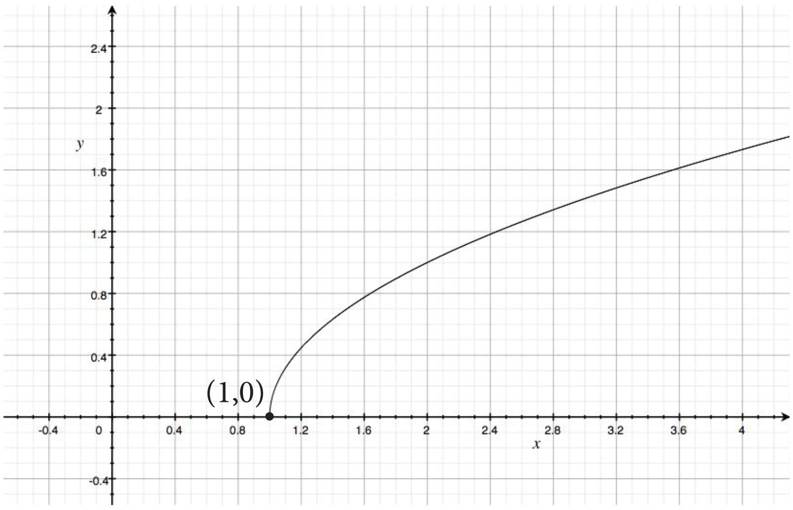


7.

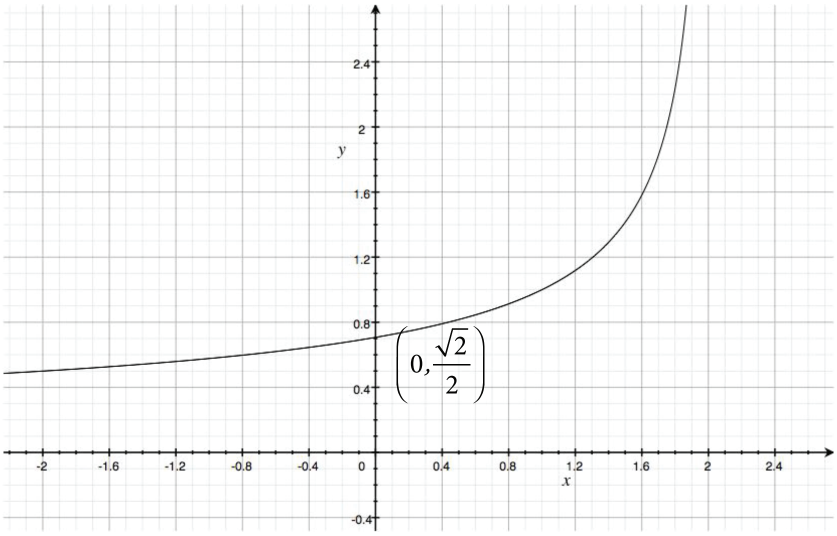


The y-axis intercept of this graph is at 0^0. The 'value' of this expression remains a topic of animated discussion amongst mathematicians. The minimum point is at approximately (0.36.0.69).

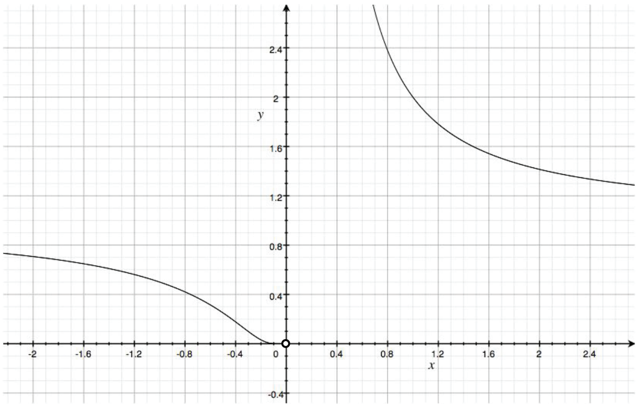
8.



9.

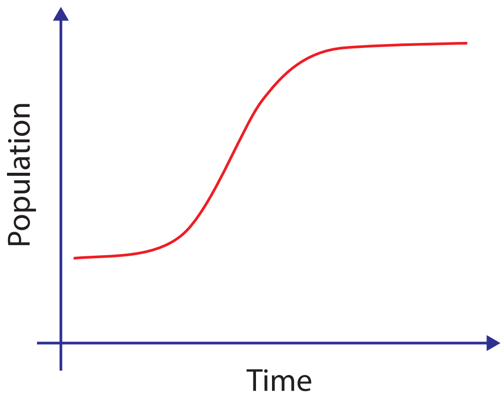


10.

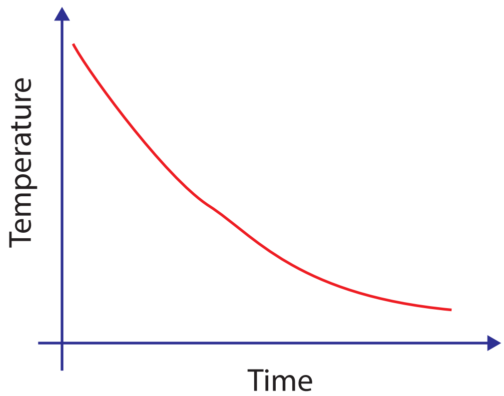


Exercise B.3.4

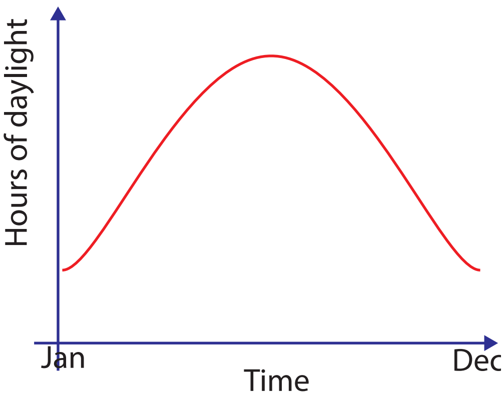
1.



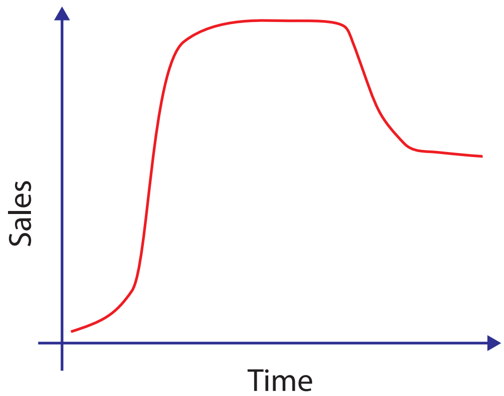
2.



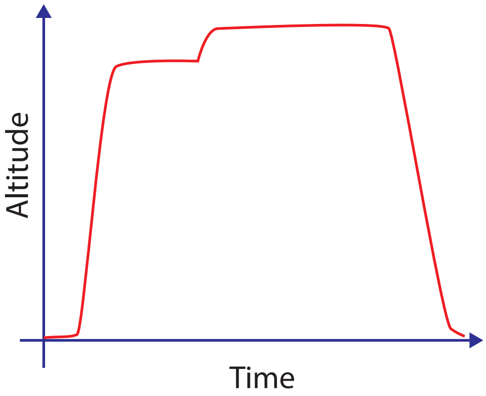
3.



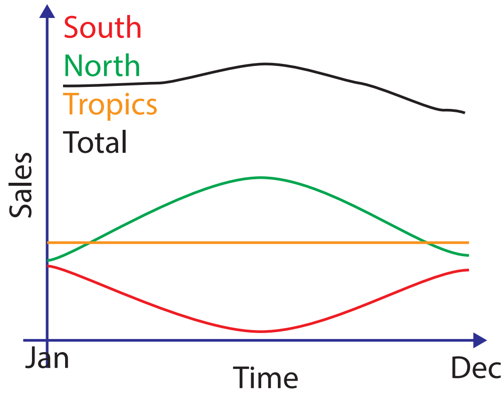
4.



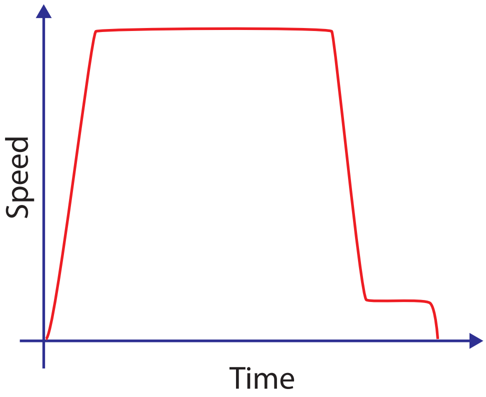
5.


Altitude
Time

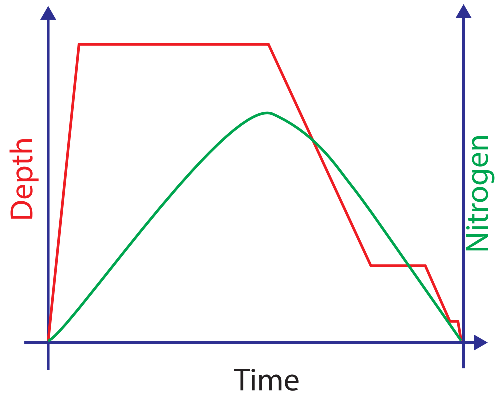
6.


South
North
Tropics
Total
Sales
Jan
Time
Dec

7.

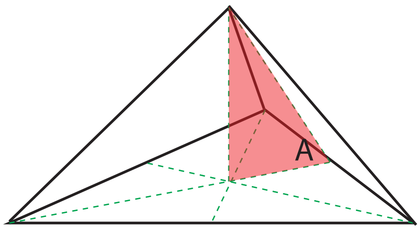

Speed
Time

8.


Depth
Nitrogen
Time

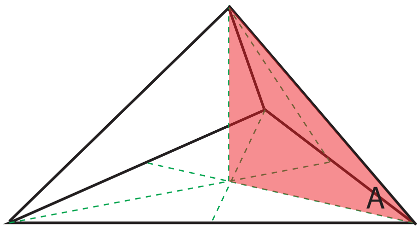
Exercise C.1.1

1.



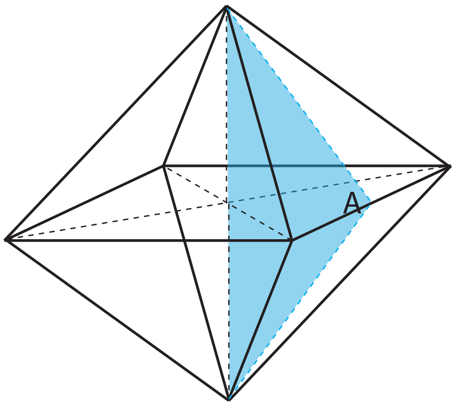
Not a unique answer.

2.



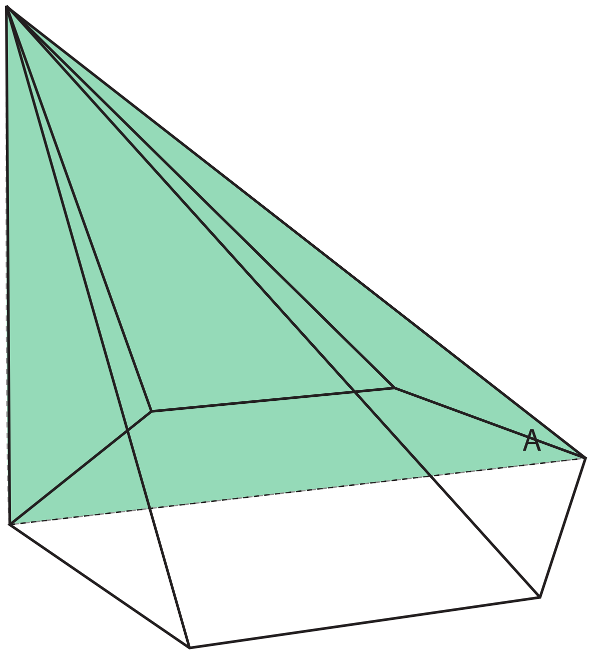
Not a unique answer.

3.



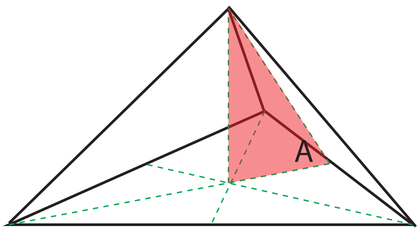
Not a unique answer.

4.

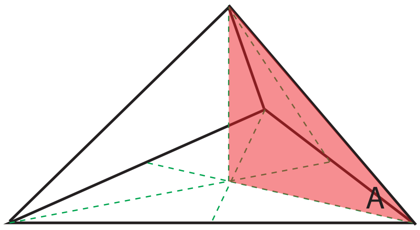


Exercise C.1.1

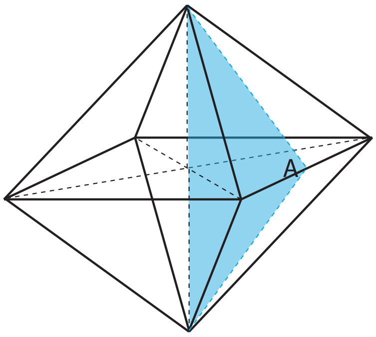
1.



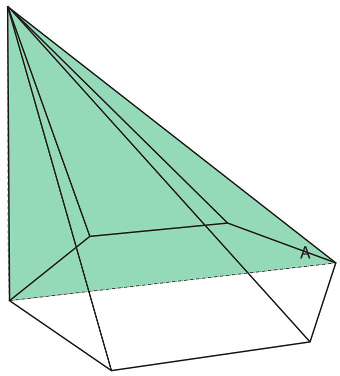
2.



3.



4.



Exercise C.1.2

1. a $V = 7500 \text{ cm}^3$ $A = 2350 \text{ cm}^2$ b $V = 10013 \text{ cm}^3$ $A = 2878 \text{ cm}^2$

   c $V = 72.726 \text{ cm}^3$ $A = 115.22 \text{ cm}^2$ d $V = 0.03564 \text{ cm}^3$ $A = 1.9572 \text{ cm}^2$

2. a $V = 354 \text{ cm}^3$ $A = 279 \text{ cm}^2$ b $V = 1318 \text{ cm}^3$ $A = 1106 \text{ cm}^2$

   c $V = 1912 \text{ cm}^3$ $A = 222 \text{ cm}^2$ d $V = 155 \text{ cm}^3$ $A = 111 \text{ cm}^2$

3. a $V = 3 \text{ cm}^3$ $A = 13 \text{ cm}^2$ b $V = 3.5 \text{ cm}^3$ $A = 24.9 \text{ cm}^2$

   c $V = 374.8 \text{ cm}^3$ $A = 467 \text{ cm}^2$

   d $V = 567\,581 \text{ mm}^3$ (inner bowl holds 15.3 l) $A = 88\,592 \text{ mm}^2$

   e $V = 3\,000 \text{ mm}^3$ $A = 1\,383 \text{ mm}^2$ f $V = 288.7 \text{ mm}^3$ $A = 552 \text{ mm}^2$

   g $V = 138.6 \text{ cm}^3$ $A = 166.3 \text{ cm}^2$. Note that this follows from Example 3.1.1.

   h $V = 3\,741 \text{ mm}^3$ $A = 1\,314 \text{ mm}^2$

4. a About 150 mm b ~80 mm

   c ~186 mm

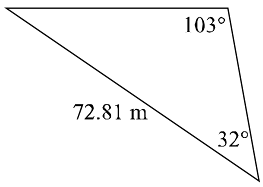
5. $V = 25.28 \text{ m}^3$

6. $\dfrac{\sqrt{2}a^3}{12}$

7. $\dfrac{15+7\sqrt{5}}{4}a^3$

Exercise C.2.8

6. A parallelogram has sides of length 21.90 cm and 95.18 cm. The angle between these sides is 121°. Find the length of the long diagonal of the parallelogram.

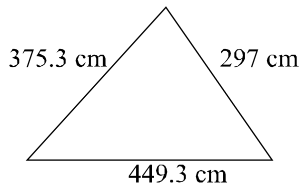


7. A town clock has 'hands' that are of length 62cm and 85cm.

   a Find the angle between the hands at half past ten.

   b Find the distance between the tips of the hands at half past ten.

8. A shop sign is to be made in the shape of a triangle. The lengths of the edges are shown. Find the angles at the vertices of the sign.

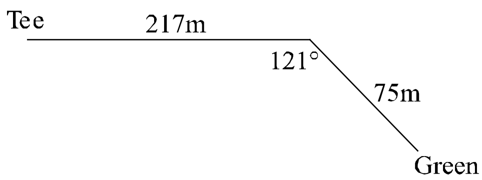


9. An aircraft takes off from an airstrip and then flies for 16.2 km on a bearing of 066°T. The pilot then makes a left turn of 88° and flies for a further 39.51 km on this course before deciding to return to the airstrip.

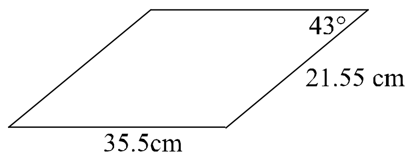
   a Through what angle must the pilot turn to return to the airstrip?

   b How far will the pilot have to fly to return to the airstrip?

10. A golfer hits two shots from the tee to the green. How far is the tee from the green?



11. The diagram shows a parallelogram. Find the length of the longer of the two diagonals.



12. A triangle has angles 64°, 15° and 101°. The shortest side is 49 metres long. What is the length of the longest side?

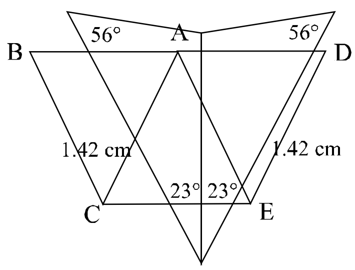
13. The diagram shows a part of the support structure for a tower. The main parts are two identical triangles, ABC and ADE.

    AC = DE = 27.4cm and BC = AE = 23.91cm

    The angles ACB and AED are 58°.

    Find the distance BD.

14. The diagram shows a design for the frame of a piece of jewellery. The frame is made of wire.



Find the length of wire needed to make the frame.

15. A triangular cross-country running track begins with the runners running North for 2050 metres. The runners then turn right and run for 5341 metres on a bearing of 083°T. Finally, the runners make a turn to the right and run directly back to the starting point.

a Find the length of the final leg of the run.

b Find the total distance of the run.

c What is the angle through which the runners must turn to start the final leg of the race?

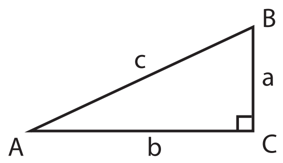
d Find the bearing that the runners must take on the final leg of the race.

16 Show that for any standard triangle ABC,

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

Exercise C.2.1

1.	a	b	c	A	B	C
a	13.3	37.1	48.2	10°	29°	141°
b	2.7	1.2	2.8	74°	25°	81°
c	11.0	0.7	11.3	60°	3°	117°
d	31.9	39.1	51.7	38°	49°	93°
e	18.5	11.4	19.5	68°	35°	77°
f	14.6	15.0	5.3	75°	84°	21°
g	26.0	7.3	26.4	79°	16°	85°



2. a $2\sqrt{3}$ b $5\left(1+\sqrt{3}\right)$ c 4 d $2\left(1+\sqrt{3}\right)$

e $\frac{4}{3}\left(3+\sqrt{3}\right)$ f $\sqrt{106}-5$

4. a $25\left(1+\sqrt{3}\right)$ b $\frac{40\sqrt{3}}{3}$

Exercise C.2.2

	a cm	b cm	c cm	A	B	C
1	13.3	37.1	48.2	10°	29°	141°
2	2.7	1.2	2.8	74°	25°	81°
3	11.0	0.7	11.3	60°	3°	117°
4	31.9	39.1	51.7	38°	49°	93°
5	18.5	11.4	19.5	68°	35°	77°
6	14.6	15.0	5.3	75°	84°	21°
7	26.0	7.3	26.4	79°	16°	85°
8	21.6	10.1	28.5	39°	17°	124°
9	0.8	0.2	0.8	82°	16°	82°
10	27.7	7.4	33.3	36°	9°	135°
11	16.4	20.7	14.5	52°	84°	44°
12	21.4	45.6	64.3	11°	24°	145°
13	30.9	27.7	22.6	75°	60°	45°
14	29.3	45.6	59.1	29°	49°	102°
15	9.7	9.8	7.9	65°	67°	48°
16	21.5	36.6	54.2	16°	28°	136°
17	14.8	29.3	27.2	30°	83°	67°
18	10.5	0.7	10.9	52°	3°	125°
19	11.2	6.9	17.0	25°	15°	140°
20	25.8	18.5	40.1	30°	21°	129

Exercise C.2.3

	a	b	c	A°	B°	C°	c*	B*°	C*°
1	7.40	18.10	21.06	20.00	56.78	103.22	12.95	123.22	36.78
2	13.30	19.50	31.36	14.00	20.77	145.23	6.49	159.23	6.77
3	13.50	17.00	25.90	28.00	36.24	115.76	4.12	143.76	8.24
4	10.20	17.00	25.62	15.00	25.55	139.45	7.22	154.45	10.55
5	7.40	15.20	19.55	20.00	44.63	115.37	9.02	135.37	24.63
6	10.70	14.10	21.41	26.00	35.29	118.71	3.94	144.71	9.29
7	11.50	12.60	22.94	17.00	18.68	144.32	1.16	161.32	1.68
8	8.30	13.70	18.67	24.00	42.17	113.83	6.36	137.83	18.17
9	13.70	17.80	30.28	14.00	18.32	147.68	4.27	161.68	4.32

10	13.40	17.80	26.19	28.00	38.58	113.42	5.24	141.42	10.58
11	12.10	16.80	25.63	23.00	32.85	124.15	5.30	147.15	9.85
12	12.00	14.50	24.35	21.00	25.66	133.34	2.72	154.34	4.66
13	12.10	19.20	29.34	16.00	25.94	138.06	7.57	154.06	9.94
14	7.20	13.10	19.01	15.00	28.09	136.91	6.30	151.91	13.09
15	12.20	17.70	23.73	30.00	46.50	103.50	6.93	133.50	16.50
16	9.20	20.90	27.97	14.00	33.34	132.66	12.59	146.66	19.34
17	10.50	13.30	21.96	20.00	25.67	134.33	3.03	154.33	5.67
18	9.20	19.20	26.29	15.00	32.69	132.31	10.80	147.31	17.69
19	7.20	13.30	18.33	19.00	36.97	124.03	6.82	143.03	17.97
20	13.50	20.40	25.96	31.00	51.10	97.90	9.01	128.90	20.10

21 a–d no triangles exist.

Exercise C.2.4

1 30.64 km

2 4.57 m

3 476.4 m

4 201°47'T

5 222.9 m **a** 3.40 m **b** 3.11 m

6 **b** 1.000 m **c** 1.715 m

7 **a** 51.19 min **b** 1 hr 15.96 min **c** 14.08 km

8 $4886

9 906 m

Exercise C.2.5

a cm	b cm	c cm	A	B	C	
1	13.5	9.8	16.7	54°	36°	90°
2	8.9	10.8	15.2	35°	44°	101°
3	22.8	25.6	12.8	63°	87°	30°
4	21.1	4.4	21.0	85°	12°	83°
5	15.9	10.6	15.1	74°	40°	66°
6	8.8	13.6	20.3	20°	32°	128°
7	9.2	9.5	13.2	44°	46°	90°
8	23.4	62.5	58.4	22°	89°	69°
9	10.5	9.6	15.7	41°	37°	102°
10	21.7	36.0	36.2	35°	72°	73°
11	7.6	3.4	9.4	49°	20°	111°
12	7.2	15.2	14.3	28°	83°	69°
13	9.1	12.5	15.8	35°	52°	93°
14	14.9	11.2	16.2	63°	42°	75°
15	2.0	0.7	2.5	38°	13°	129°
16	7.6	3.7	9.0	56°	24°	100°
17	18.5	9.8	24.1	45°	22°	113°
18	20.7	16.3	13.6	87°	52°	41°

19	14.6	22.4	29.9	28°	46°	106°
20	7.0	6.6	9.9	45°	42°	93°
21	21.8	20.8	23.8	58°	54°	68°
22	1.1	1.7	1.3	41°	89°	50°
23	1.2	1.2	0.4	85°	76°	19°
24	23.7	27.2	29.7	49°	60°	71°
25	3.4	4.6	5.2	40°	60°	80°

Exercise C.2.6

1 **a** 10.14 km **b** 121°T

2 7° 33’

3 4.12 cm

4 57.32 m

5 315.5 m

6 **a** 124.3 km **b** W28° 47’ S

Exercise C.2.7

1

a	1999.2 cm^2	**b**	756.8 cm^2	**c**	3854.8 cm^2	**d**	2704.9 cm^2
e	538.0 cm^2	**f**	417.5 cm^2	**g**	549.4 cm^2	**h**	14.2 cm^2
i	516.2 cm^2	**j**	281.5 cm^2	**k**	918.8 cm^2	**l**	387.2 cm^2
m	139.0 cm^2	**n**	853.7 cm^2	**o**	314.6 cm^2		

2 69 345 m^2

3 $100\pi - 6\sqrt{91}$ cm^2

4 17.34 cm

5 **a** 36.77sq units **b** 14.70 sq units **c** 62.53 sq units

6 52.16 cm^2

7 7° 2’

8 $\frac{(b + a \times \tan\theta)^2}{2\tan\theta}$

9 Area of $\Delta ACD = 101.78$ cm^2 , Area of $\Delta ABC = 61.38$ cm^2

Exercise C.2.8

1 39.60 m, 52.84 m

2 30.2 m

3 54°,42°, 84°

4 37°

5 028°T

6 108.1 cm

7 a 135° b 136.1 cm

8 41°, 56°, 83°

9 a 158° left b 43.22 km

10 264 m

11 53.33 cm

12 186 m

13 50.12 cm

14 5.17 cm

15 a 5950 m b 13341 m c 160° d 243°

16 a 20.70° b 2.578 m c 1.994 m^3

Exercise C.2.9

1. a i 030°T ii 330°T iii 195°T iv 200°T

b i N25°E ii S iii S40°W

iv N10°W

2. 37.49 m

3. 18.94 m

4. 37° 18’

5. $^{26}/_{9}$ m/s

6. N58° 33’W, 37.23 km

7. 199.82 m

8. 10.58 m

9. 72.25 m

10. 25.39 km

11. 5.76 m

12. a 3.01 km N, 3.99 km E b 2.87 km E 0.88 km S c 6.86 km E 2.13 km N

d 7.19 km 253°T

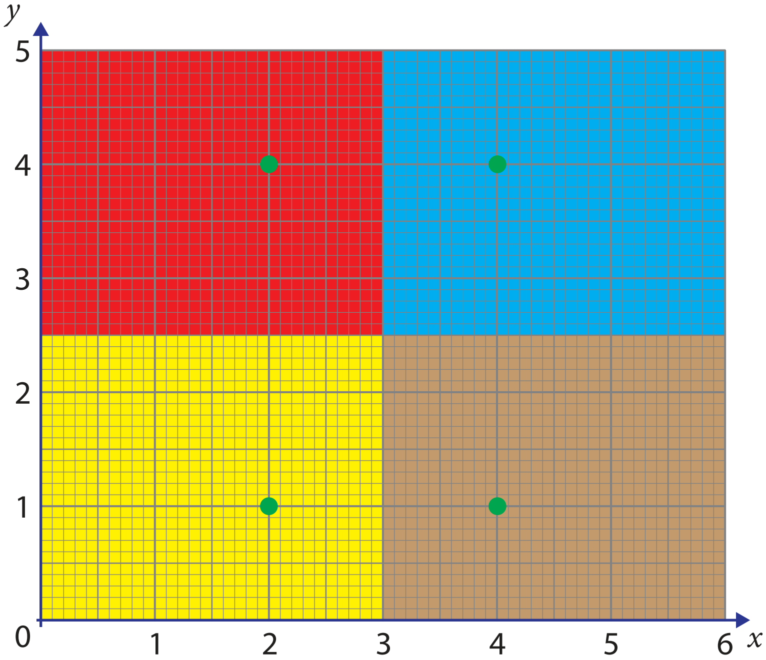
13. 524 m

Exercise C.3.1

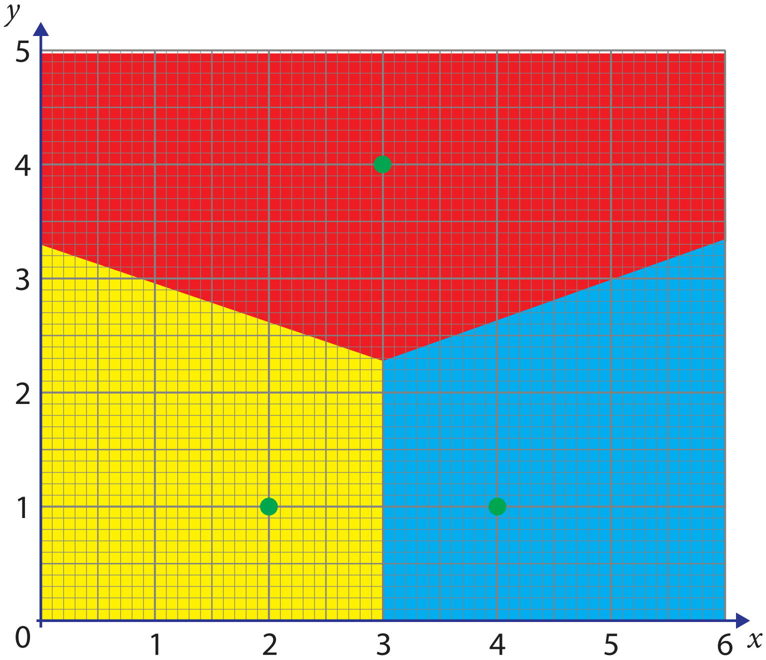
1.	a	39°48'	b	64°46'		
3.	a	21°48'	b	42°2'	c	26°34'
4.	a	2274	b	12.7°		
5.	251.29 m					
6.	a	103.52 m	b	35.26°	c	39.23°
7.	b	53.43 m	c	155.16 m	d	98.37 m
9.	a	$\sqrt{(b-c)^2+h^2}$	b	$\tan^{-1}\left(\frac{h}{a}\right)$	c	$\tan^{-1}\left(\frac{h}{b-c}\right)$
	d	$2(b+c)\sqrt{h^2+a^2}+2a\sqrt{(b-c)^2+h^2}$				
10.	82.80 m					
11.	a	40.61 m	b	49.46 m		
12.	a	10.61 cm	b	75° 58'	c	93° 22'
13.	a	1.44 m	b	73° 13'	c	62° 11'
15.	6.2°					
16.	65.76°, 14.3°					
17.	146.9m, 47.1°					
18.	711m, 29.1					

Exercise C.6.1

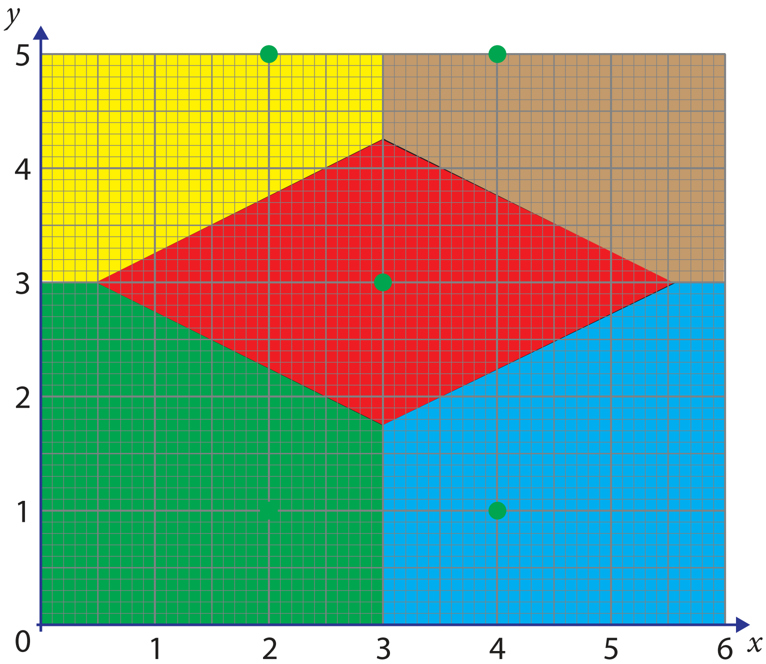
1. a



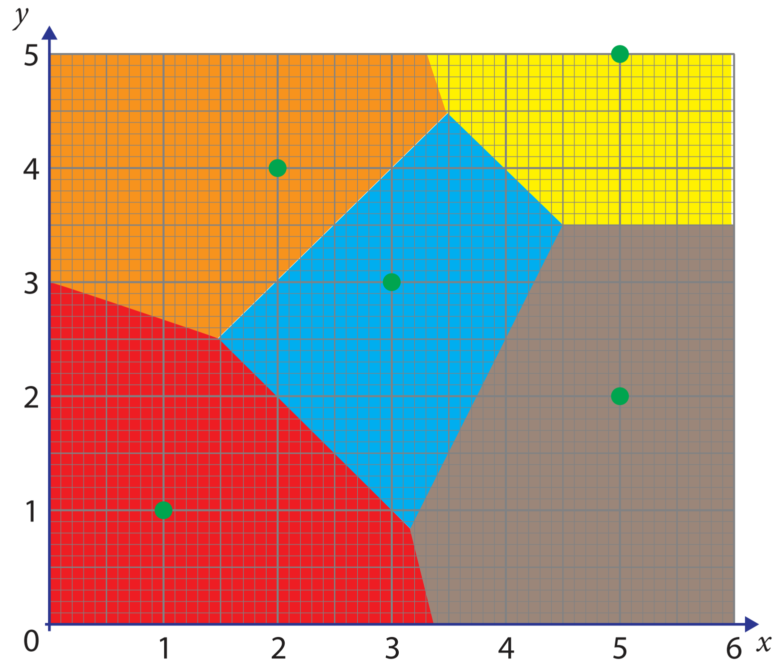
b



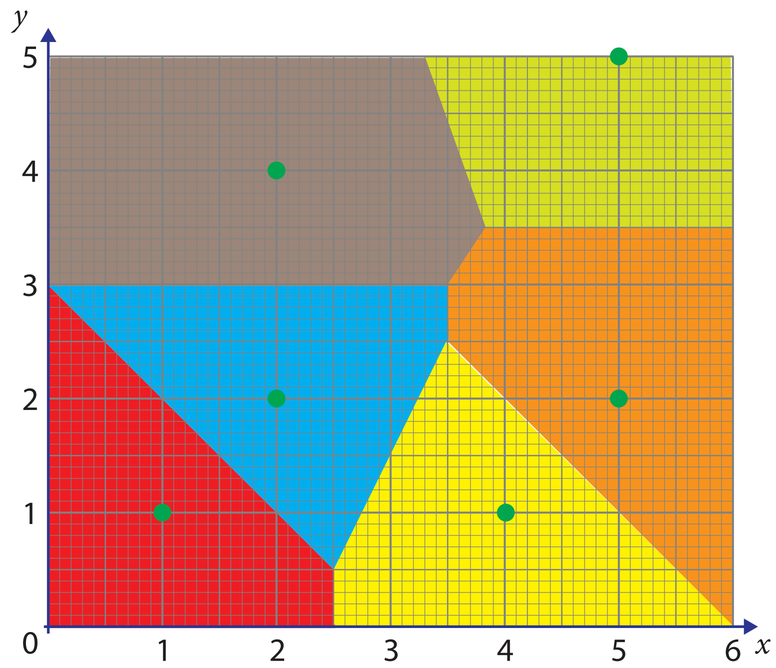
c



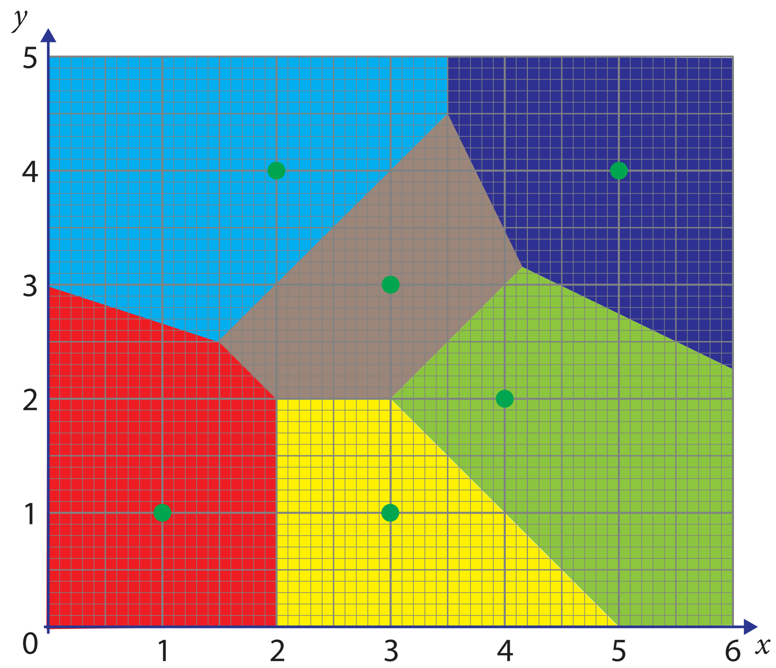
d



e



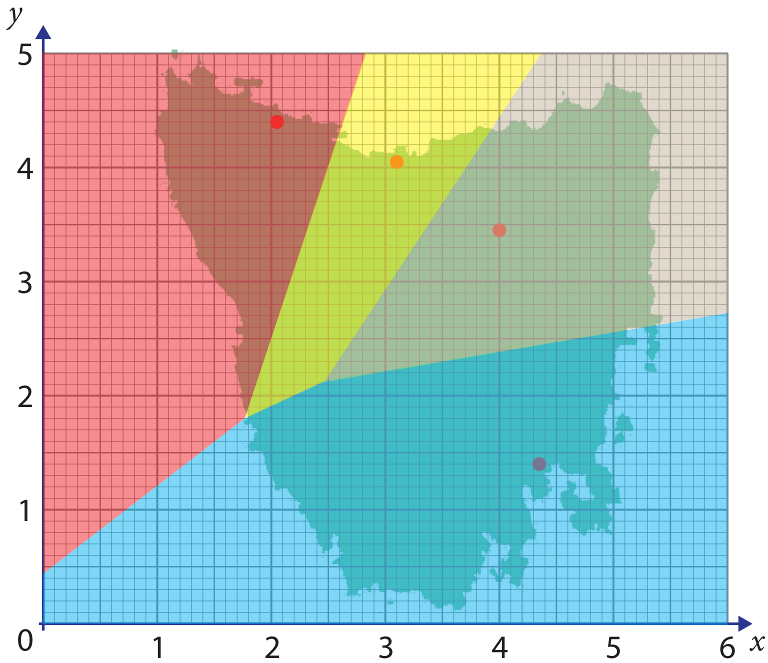
f



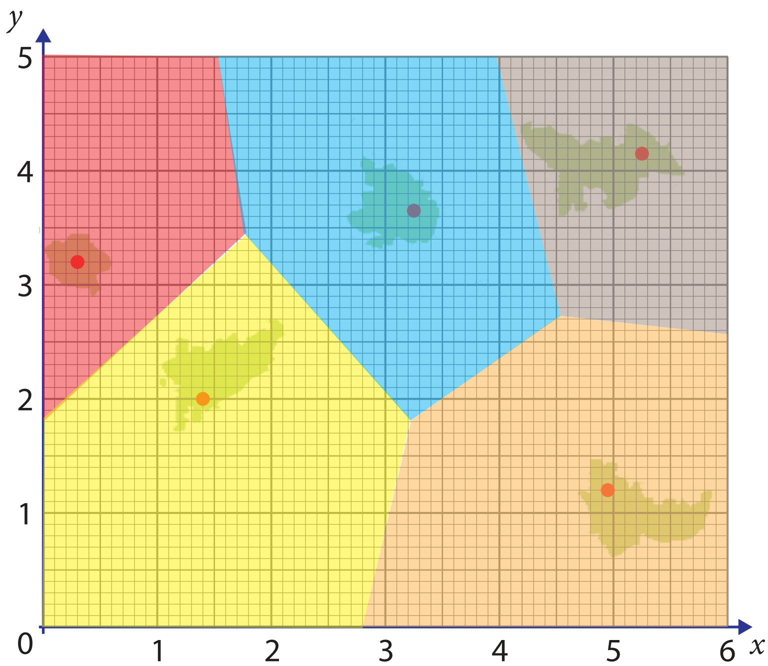
2. (3,2.5)

3. (3,2.3)

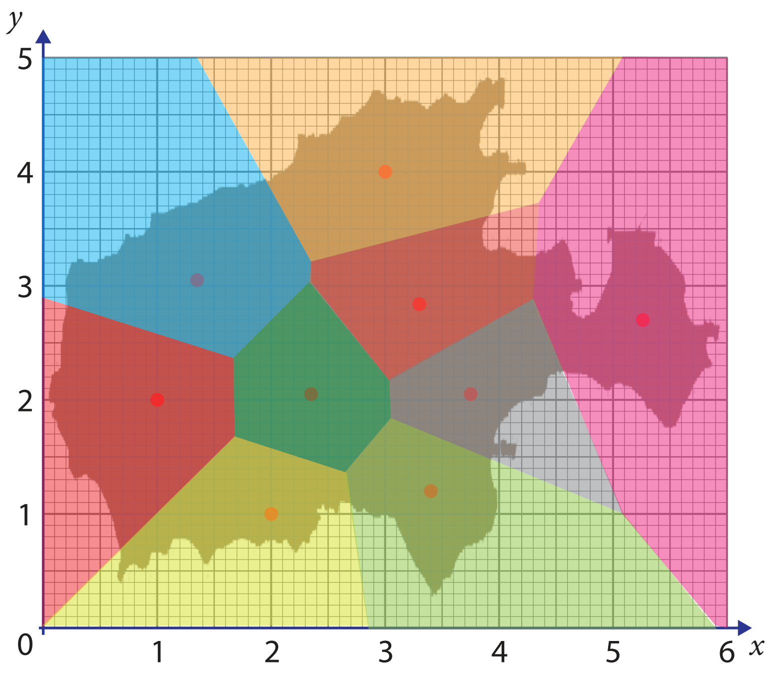
4.



5.



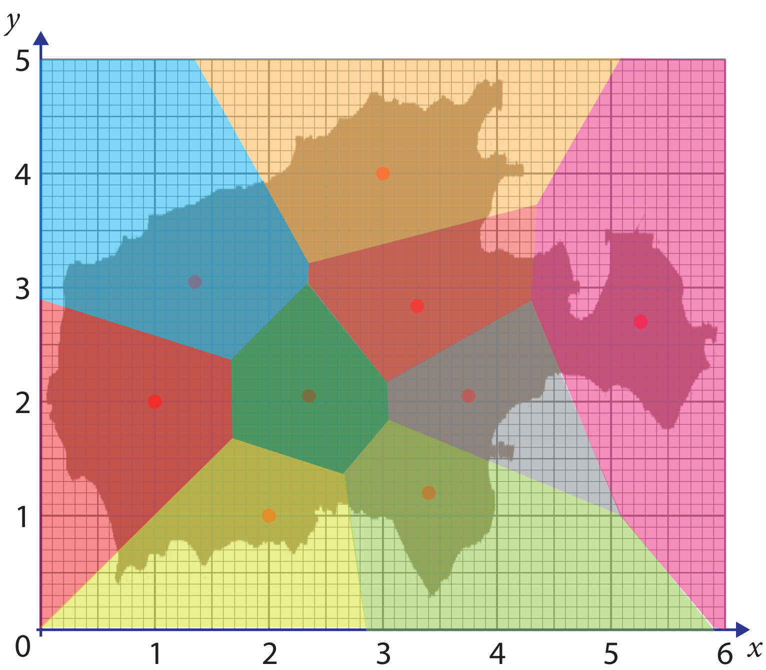
6.



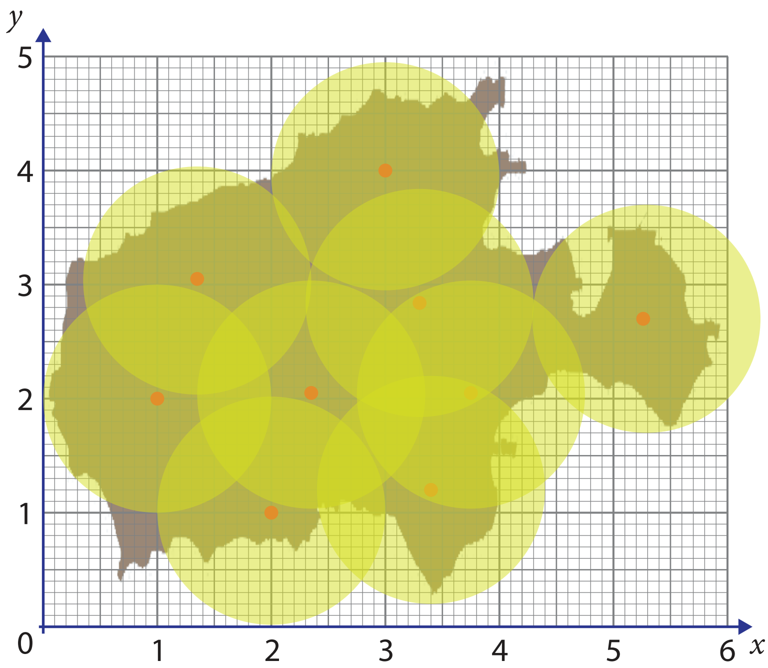
Dump at (1.75,3)

7. $\left(\frac{63}{22},\frac{45}{22}\right)$

9. a

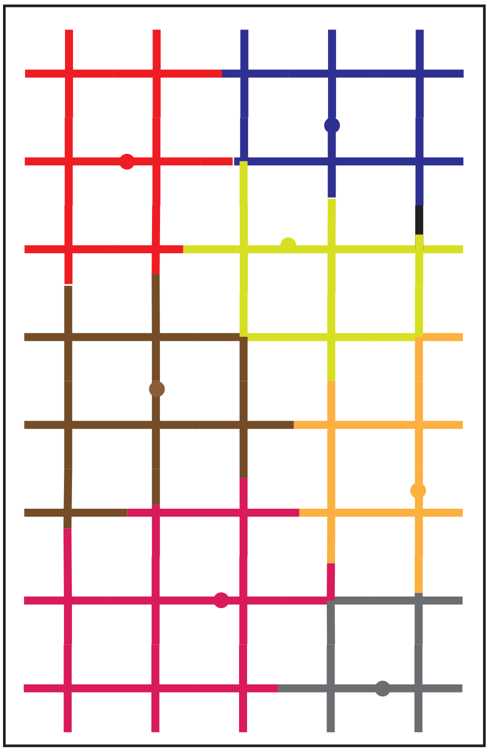


b



The coverage is good!

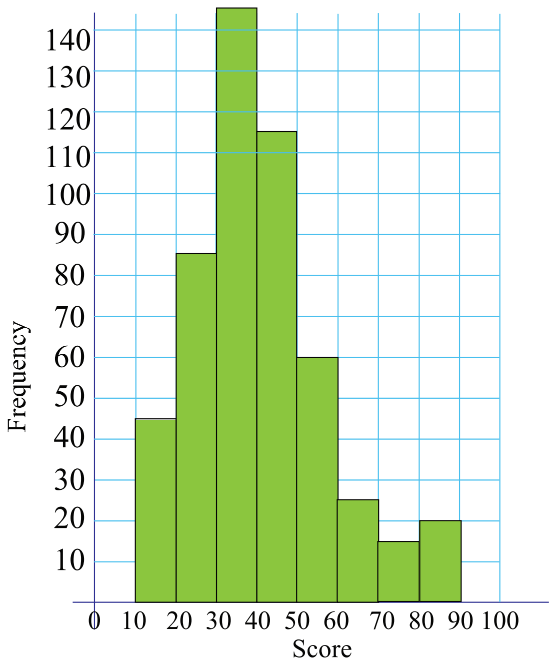
10.



Exercise D.2.1

1 **a** 53 **b** 34% **c** 9.4%

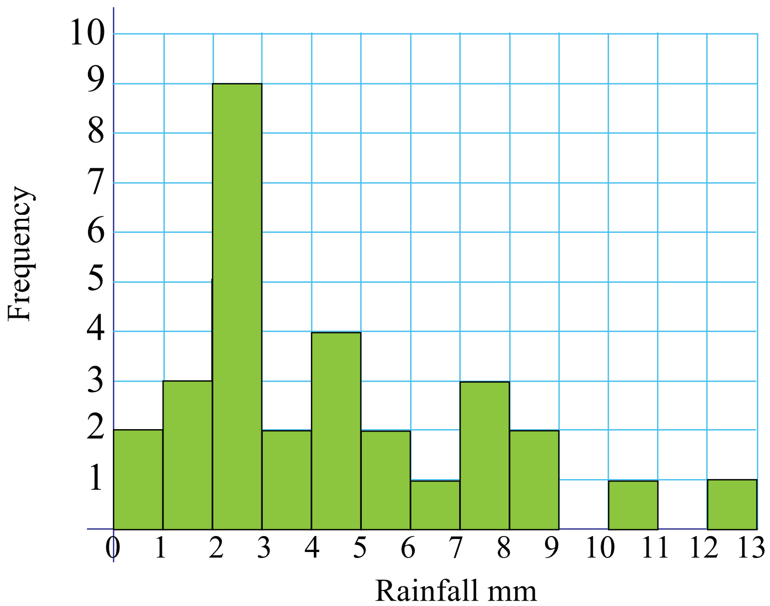
2



b 74% **c** 80

3 **a** Continuous

b



Exercise D.2.2

1 a Med = 5, Q1 = 2, Q3 = 7, IQR = 5 b Med = 3.3, Q1 = 2.8, Q3 = 5.1, IQR = 2.3

c Med = 163.5, Q1 = 143, Q3 =182, IQR = 39

d Med = 1.055, Q1 = 0.46, Q3 = 1.67, IQR = 1.21

e Med = 5143.5, Q1 = 2046, Q3 = 6252, IQR = 4206

2 a Med = 3, Q1 = 2, Q3 = 4, IQR = 2 b Med = 13, Q1 = 12, Q3 = 13, IQR = 1

c Med = 2, Q1 = 2, Q3 = 2.5, IQR = 0.5 d Med = 40, Q1 = 30, Q3 = 50, IQR = 20

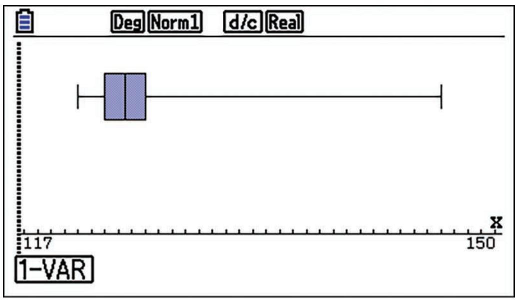
	e	Med = 20, Q1 = 15, Q3 = 22.5, IQR = 7.5						
3	a	$84.67	b	$147.8	c	$11	d	Q1 = $4.50, Q3 = $65 IQR = $60.50
	e	Median and IQR.						
4	a	2.35	b	1.25	c	2	d Q1 = 1, Q3 = 3, IQR = 2	
5	a	$232	b	$83	c–e	Med = $220, Q1 = $160, Q3 = $310, IQR = $150		
6.	Med = 14, Q1 = 10, Q3 = 19, IQR = 9							
7.	a	61%	b	86%	c	2	d	4
	e	5	f	8				

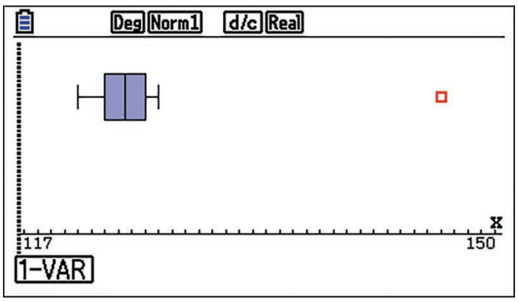
The cumulative distribution is:

Number of Errors	Number of Patients	Cumulative
0	1	1
1	3	4
2	11	15
3	28	43
4	34	77
5	14	91
6	15	106
7	23	129
8	11	140
9	7	147
10	2	149
11	1	150

Exercise D.2.3

1.





2. The value 27.36 has probably been mis-recorded an should have been 2.736. It should be discarded. Bearing in mind the errors evident in the data, the result should be reported as 2.73 gm/cc as the mean is 2.734.

3. There is no correct answer. Most donations are $5 to $25 with the median $15.

Exercise D.3.1

1. **a** Sample A Mean = 1.99 kg; Sample B Mean = 2.00 kg
   b Sample A Sample std = 0.0552 kg; Sample B Sample std = 0.1877 kg
   c Sample A Population std = 0.0547 kg; Sample B Population std = 0.1858 kg
2. **a** 16.4 **b** 6.83
3. Mean = 49.97; Std = 1.365
4. **a** $84.67 **b** $148
5. **a** 2.35 **b** 1.25
6. **a** $232 **b** $83
7. **c** 40
8. **a i** 20.17 **ii** 7.29 **b** 31 **c** 20.76
9. $\mu = 1.11$ lb, $\sigma = 0.0033$ lb
10. $\mu = 35$ mph, $\sigma = 1.25$ mph
11. $\mu = 234.6$ kg, $\sigma = 3.6$ kg.

Exercise D.4.1

1. a i Increasing, positive ii approx. linear iii mild (to weak)

   b i No association

   c i Increasing, positive ii linear iii very strong

   d i Increasing, positive ii square root

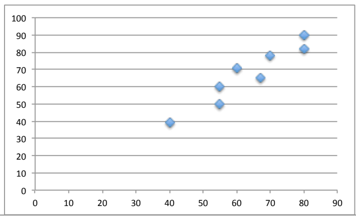
   iii mild (strength not appropriate as it is a non–linear relationship!)

   e i Decreasing, negative ii exponential

   iii mild (strength not appropriate as it is a non–linear relationship!)

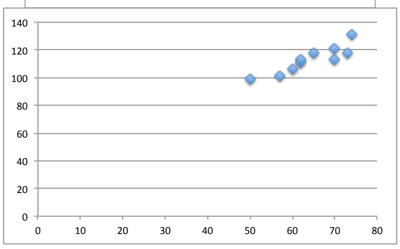
   f i Decreasing ii approx. linear iii weak

2. a



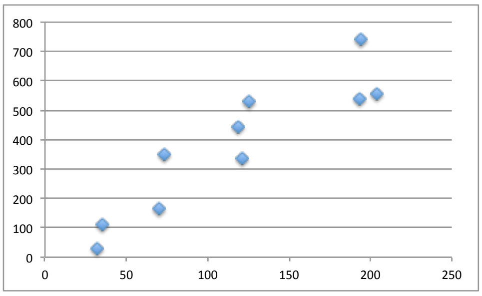
   b Positive association, linear, strength: very strong

3. a



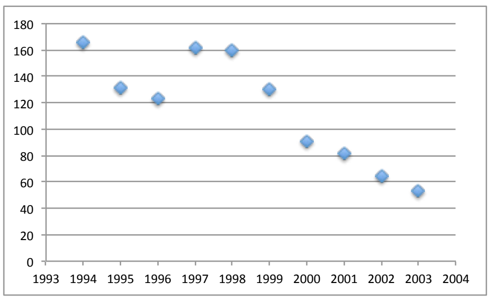
   b Positive association, linear, strength: very strong

4.



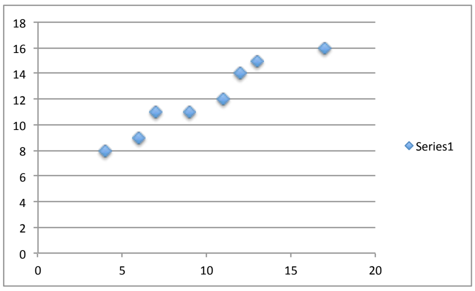
Data displays a strong positive association. Increase in lead content can be attributed to increase in traffic flow.

5.

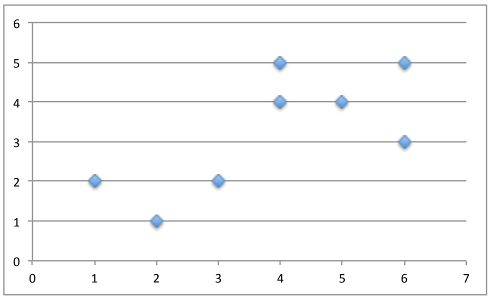


Worksafety policy has had desired effect, i.e. number of accidents has decreased. Data displays a strong negative association.

1. a $r = 0.96$ b



2. a



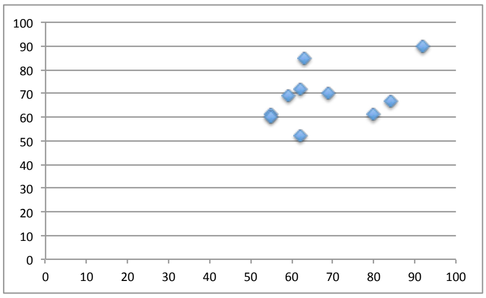
b $r = 0.70$ (aasumed linear)

3. $r = 0$. No, not linear!

4. No. The relationship is not linear.

5. a i 64% ii 81% b i 51% ii 64%

6. a



b $r = 0.45$

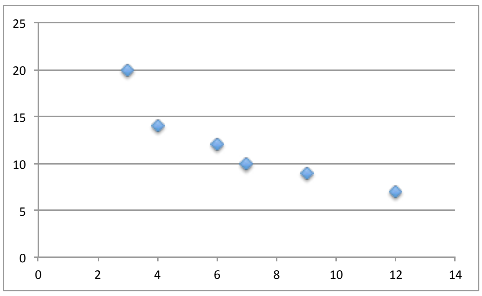
7. 3

8. ± 0.922

9. 82%

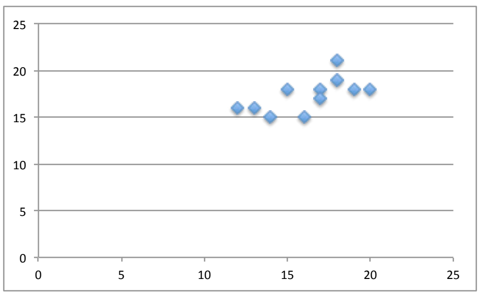
Exercise D.4.3

1. a i

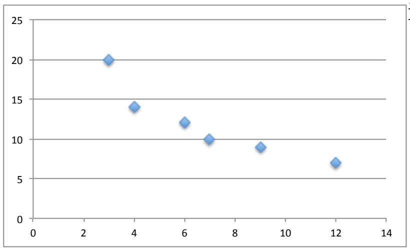


ii By eye (approx) $y = -1.33x + 21.11$

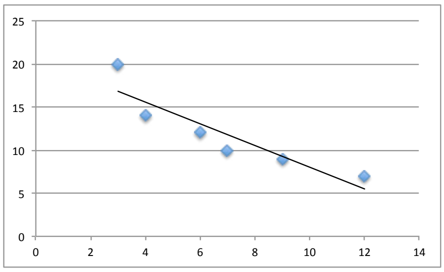
a i

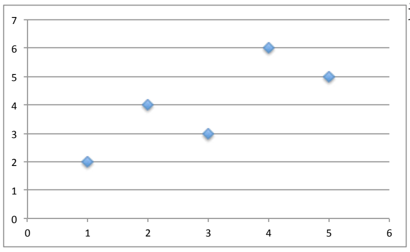


ii By eye (approx) $y = 0.64x + 6.94$

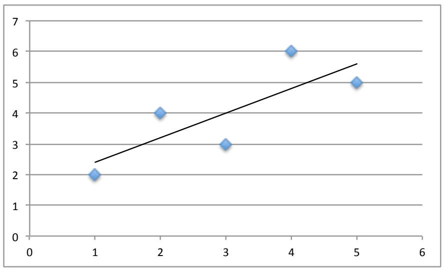
2. a i 

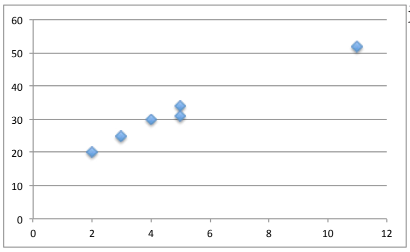
ii $y = 20.6 - 1.26x$

iii 

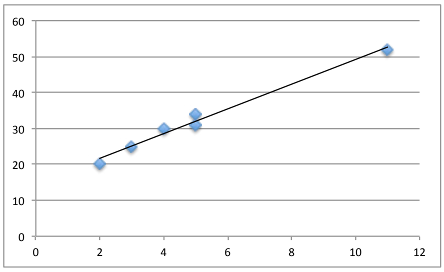
b i 

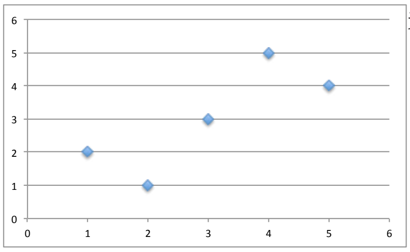
ii $y = = 1.6 + 0.8x$

iii 

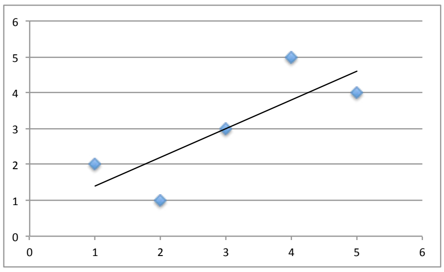
c i 

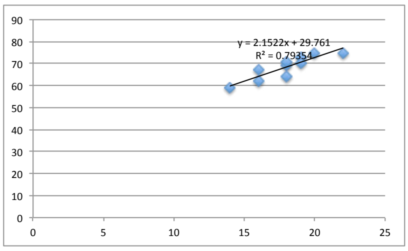
ii $y = 14.8 + 3.44x$

iii 

d i 

ii $y = 0.6 + 0.8x$

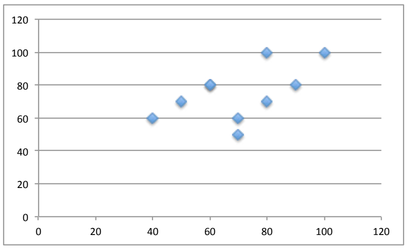
iii 

3. a 

b $r = 0.891$

c 79.4%

d $y = 29.76 + 2.15x$

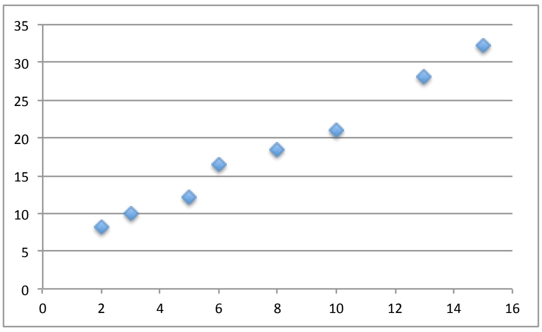
4. a 

b r = 0.553

c i $y = 40 + 0.5x$

ii $x = 24.1 + 0.61y$

5. a



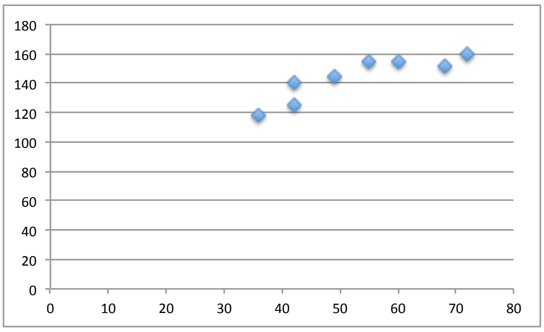
b Based on the scatter diagram, there is a definite linear relationship. Therefore, owner is justified.

c i $r = 0.99$ ii $C = 4.19 + 1.82w$

d i 20.57, i.e. 21 ii 95.19, i.e. 95

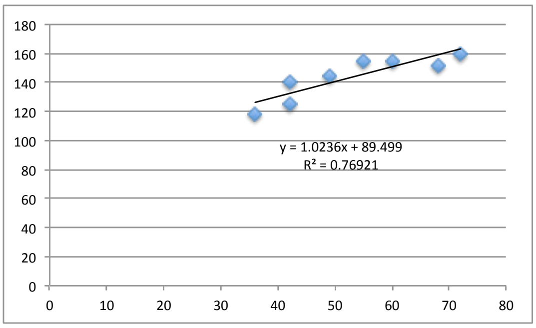
iii From ii, serving 95 people per hour is unrealistic.

6. a



b Scatter diagram shows a linear relationship. Therefore statistic is appropriate, $r = 0.877$.

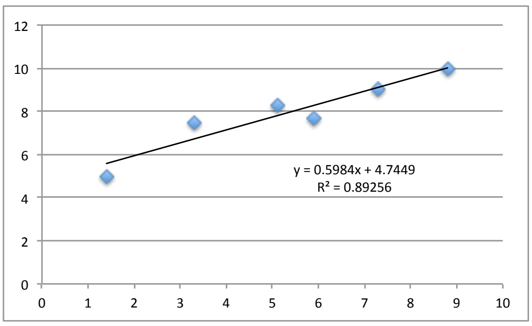
c i



d i 135.6 ii 176.5

iii $x = 85$ is a fair way out from the set of values used to obtain the regression line.

7. a-c



b Scatter diagram shows a linear relationship. Therefore statistic is appropriate, $r = 0.945$

c i $y = 4.74 + 0.6x$ ii d i 8.63 ii 10.73

Exercise D.5.3

15. Dale and Kritt are trying to solve a physics problem. The chances of solving the problem are Dale—65% and Kritt—75%. Find the probability that:

    a only Kritt solves the problem.

    b Kritt solves the problem.

    c both solve the problem.

    d Dale solves the problem given that the problem was solved.

16. A coin is weighted in such a way that there is a 70% chance of it landing heads. The coin is tossed three times in succession. Find the probability of observing:

    a three tails.

    b two heads.

    c two heads given that at least one head showed up.

Exercise D.5.4

14. A student runs the 100 m, 200 m and 400 m races at the school athletics day. He has an 80% chance of winning any one given race. Find the probability that he will:

    a win all 3 races.

    b win the first and last race only.

    c win the second race given that he wins at least two races.

15. Dale and Kritt are trying to solve a physics problem. The chances of solving the problem are Dale—65% and Kritt—75%. Find the probability that:

    a only Kritt solves the problem.

    b Kritt solves the problem.

    c both solve the problem.

    d Dale solves the problem given that the problem was solved.

16. A coin is weighted in such a way that there is a 70% chance of it landing heads. The coin is tossed three times in succession. Find the probability of observing:

    a three tails.

    b two heads.

    c two heads given that at least one head showed up.

Exercise 5.5.1

1 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{2}{5}$

2 **a** $\frac{2}{7}$ **b** $\frac{5}{7}$

3 **a** $\frac{5}{26}$ **b** $\frac{21}{26}$

4 {HH, HT, TH, TT} **a** $\frac{1}{4}$ **b** $\frac{3}{4}$

5 {HHH,HHT,HTH,THH,TTT,TTH,THT,HTT} **a** $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

6 **a** $\frac{2}{9}$ **b** $\frac{2}{9}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$

7 **a** $\frac{1}{2}$ **b** $\frac{3}{10}$ **c** $\frac{9}{20}$

8 **a** $\frac{11}{36}$ **b** $\frac{1}{18}$ **c** $\frac{1}{6}$ **d** $\frac{5}{36}$

9 {GGG, GGB. GBG, BGG, BBB, BBG, BGB, GBB} **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$

10 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$

11 **a** $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{4}$

12 **a** {(1, H),(2, H),(3, H),(4,H),(5, H),(6, H),(1, T),(2, T),(3, T),(4, T),(5, T),(6,T)}
b $\frac{1}{4}$

13 **a** $\frac{1}{216}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$

Exercise D.5.2

1 **a** $\frac{1}{4}$ **b** $\frac{5}{8}$ **c** $\frac{3}{4}$

2 **a** $\frac{1}{13}$ **b** $\frac{1}{2}$ **c** $\frac{1}{26}$ **d** $\frac{7}{13}$

3 $\frac{9}{26}$

4 **a** 1.0 **b** 0.3 **c** 0.5

5 **a** 0.65 **b** 0.70 **c** 0.65

6 **a** 0.95 **b** 0.05 **c** 0.80

7 **a** {TTT,TTH,THT,HTT,HHH,HHT,HTH,THH} **b i** $\frac{3}{8}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$ **iv** $\frac{3}{8}$

8 **a** $\frac{6}{25}$ **b** $\frac{6}{25}$ **c** $\frac{13}{25}$

9 **b** $\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$ **e** $\frac{7}{12}$

10 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{8}{13}$ **d** $\frac{7}{13}$

11 **a** 0.1399 **b i** 0.8797 **ii** 0.6

12 **b** $\frac{4}{15}$ **c** $\frac{4}{15}$ **d** $\frac{11}{15}$

Exercise D.5.3

1 **a** $\frac{5}{126}$ **b** $\frac{5}{18}$ **c** $\frac{1}{126}$

2 **a** $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{2}{5}$ **d** $\frac{3}{5}$

3 **a** $\frac{72}{5525}$ **b** $\frac{1}{5525}$ **c** $\frac{1}{1201}$

4 $\frac{2}{5}$

5 **a** $\frac{63}{143}$ **b** $\frac{133}{143}$

6 **a** $\frac{5}{12}$ **b** $\frac{5}{33}$ **c** $\frac{5}{6}$

7 $\frac{3}{11}$

8 **a** $\frac{4}{13}$ **b** $\frac{9}{13}$

9 **a** $\frac{67}{91}$ **b** $\frac{22}{91}$

10 **a** $\frac{1}{4}$ **b** $\frac{1}{28}$ **c** $\frac{5}{14}$

11 **a** $\frac{5}{28}$ **b** $\frac{1}{28}$

12 $\frac{6}{13}$

13 **a** $\frac{1}{6}$ **b** $\frac{1}{4}$

14 **a** $\frac{1}{210}$ **b** $\frac{7}{9}$

15 **a** $\frac{7}{1938}$ **b** 0.6

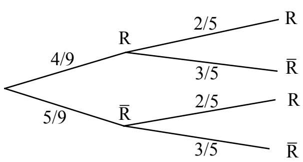
16 $\frac{11}{21}$

Exercise D.5.4

1 **a** 0.7 **b** 0.75 **c** 0.50 **d** 0.5

2 **a** 0.5 **b** 0.83 **c** 0.10 **d** 0.90

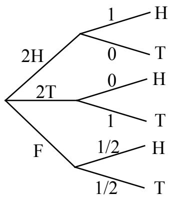
3 **a**



b $\frac{8}{45}$ **c** $\frac{22}{45}$ **d** $\frac{6}{11}$

4 **a** 0.5 **b** 0.30 **c** 0.25

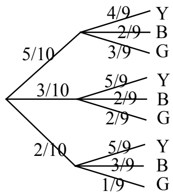
5 **a**



b $\frac{1}{2}$ **c** $\frac{2}{3}$

6 $\frac{1}{3}$

7 **a**



b $\frac{31}{45}$ **c** $\frac{2}{9}$

8 $\frac{2}{3}$

9	**a**	0.88	**b**	0.42	**c**	0.6	**d**	0.28
10	**a**	0.33	**b**	0.49	**c**	0.82	**d**	0.551
11	**a**	0.22	**b**	0.985	**c**	0.8629		
12	**a**	0.44	**b**	0.733				
14	**a**	0.512	**b**	0.128	**c**	0.8571		
15	**a**	0.2625	**b**	0.75	**c**	0.4875	**d**	0.7123
16	**a**	0.027	**b**	0.441	**c**	0.453		

Exercise D.6.1

1

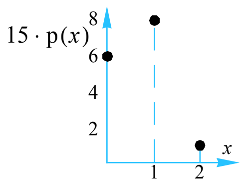
x	0	1	2	3	4
$P(x)$	0	0.1	0.2	0.5	0.2

3 0.3

4 **a** 0.1 **bi** 0.2 **ii** 0.7

5 **a** $p(0) = \frac{6}{15}, p(1) = \frac{8}{15}, p(2) = \frac{1}{15}$

b



c $\frac{14}{15}$

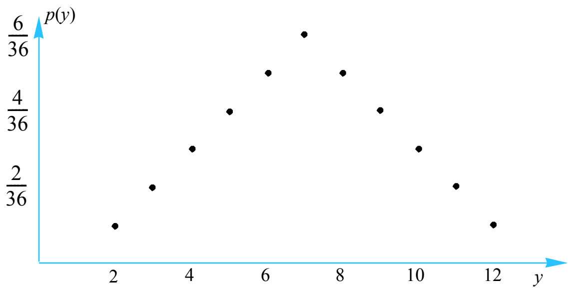
6 **a** {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

b

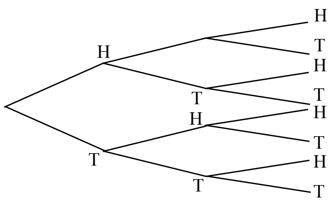
x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

c $\frac{5}{36}$

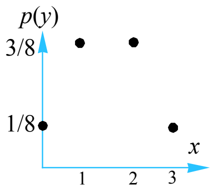
d



7 **a**



b



$p(0) = \frac{1}{8}$

$p(1) = \frac{3}{8}$

$p(2) = \frac{3}{8}$

$p(3) = \frac{1}{8}$

c $\frac{4}{7}$

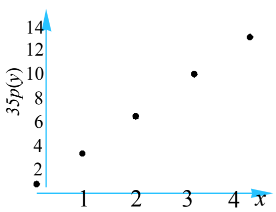
8 **a** $\frac{1}{35}$

bi $p(0) = \frac{1}{35}$ $p(1) = \frac{4}{35}$

$p(2) = \frac{7}{35}$ $p(3) = \frac{10}{35}$

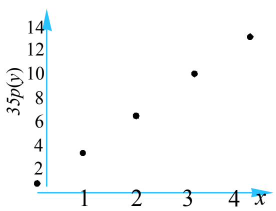
$p(4) = \frac{13}{35}$

ii



c $\frac{6}{7}$

9



a i 0.9048 **ii** 0.09048 **b** 0.0002

10 0.3712

11 **a** $p(0) = \frac{11}{30}, p(-1) = \frac{1}{2}, p(3) = \frac{2}{15}$ **b i** $\frac{11}{30}$ **ii** $\frac{13}{15}$

12

n	0	1	2
P(N = n)	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

13 **a**

n	1	2	3	4
$P(N = n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b

s	2	3	4	5	6	7	8
P(S = s)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

14 **a** 0.81 **b** 0.2439

Exercise D.6.2

1 **a** 2.8 **b** 1.86

2 **a** 3 **b i** 1 **ii** 1 **c i** 6 **ii** 0.4

3 **a i** 1.3 **ii** 2.5 **iii** –0.1

b i 0.9 **ii** 7.29 **c i** $\frac{31}{60}$ **ii** 0.3222

4 $\mu = \frac{2}{3}, \sigma^2 = 0.3556$

5 **a** 7 **b** 5.8333

6 $np = 3 \times \frac{1}{2} = 1.5$

7 **a** $\frac{1}{25}$ **b** 2.8 **c** 1.166

8 **a** 0.1 **b i** 0.3 **ii** 1

c i 0 **ii** 1 **iii** 2

9 5.56

10 $p(0) = \frac{35}{120}, p(1) = \frac{63}{120}, p(2) = \frac{21}{120}, p(3) = \frac{1}{120}$ **b i** 0.9 **ii** 0.49

c $W = 3N - 3$, $E(W) = -0.3$

11	**a**	\$ –1.00	**b**	both the same				
12	**a**	50	**b**	18	**c**	2		
13	**a**	11	**b**	$\frac{\sqrt{3}}{3}$	**c**	–4		
14	**a**	0.75	**b**	0.6339				

15 **a** $E(X) = 1 - 2p$, $Var(X) = 4p(1-p)$ **b i** $n(1-2p)$ **ii** $4np(1-p)$

16 **a**

n	0	1	2
P(N = n)	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

b $W = 21.43$

17 **a** $a = \frac{2}{3}, 0 \le b \le 1$ **b** $E(X) = \frac{b+1}{3}$, $Var(X) = \frac{1}{9}(2 + 7b - b^2)$

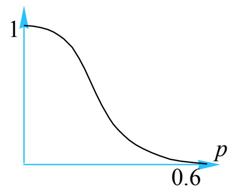
18 **a** $E(X) = 4$, $Var(X) = 20$

Exercise D.6.3

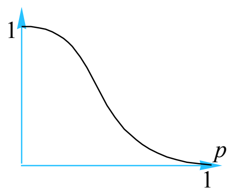
1	**a**	0.2322	**b**	0.1737	**c**	0.5941		
2	**a**	0.3292	**b**	0.8683	**c**	0.2099	**d**	0.1317
3	**a**	0.1526	**b**	0.4812	**c**	0.5678		
4	**a**	0.7738	**b**	3.125×10^{-7}	**c**	0.9988	**d**	3×10^{-5}
5	**a**	0.2787	**b**	0.4059				
6	**a**	0.2610	**b**	0.9923				
7	**a**	0.2786	**b**	0.7064	**c**	0.1061		
8	**a**	0.1318	**b**	0.8484	**c**	0.054	**d**	0.326
9	**a**	0.238	**b**	0.6531	**c**	0.0027	**d**	0.726
	e	12.86						
10	**a**	0.003	**b**	0.2734	**c**	0.6367	**d**	0.648
11	**a**	0.3125	**b**	0.0156	**c**	0.3438	**d**	3
12	**a**	0.2785	**b**	0.3417	**c**	120		
13	**a**	0.0331	**b**	0.565				
14	**a**	0.4305	**b**	0.61	**c**	\$720	**d**	0.2059
15	**a i**	1.4	**ii**	1	**iii**	1.058	**iv**	0.0795
	v	0.0047						

	b i	3.04	**ii**	3	**iii**	1.373	**iv**	0.2670
	v	0.1390						
16	38.23							
19	**a i**	0.1074	**ii**	7.9×10^{-4}	**iii**	0.3758	**b**	at least 6
20	**a**	$\frac{4}{3}$	**b**	$\frac{10}{9}$	**c**	$\frac{1}{6}$	**d**	$\frac{5}{288}$
21	**a**	20	**b**	3.4641				
22	**a**	102.6	**b**	0.000254				
23	**a i**	6	**ii**	2.4	**b i**	6	**ii**	3.6
24	0.1797							
25	1.6, 1.472							
26	**a** 0.1841 **b** \$11.93							
27	**a** \$8 **b** \$160							

28 **a** 0.0702 **c**



29 **b**



30 **b** 0.8035 **c** 39.3

31 **a** $\dfrac{np - np(1-p)^{n-1}}{1-(1-p)^n - np(1-p)^{n-1}}, 0 < p < 1$

Exercise D.6.4

1	**a**	0.6915	**b**	0.9671	**c**	0.9474	**d**	0.9965
	e	0.9756	**f**	0.0054				
2	**a**	0.0360	**b**	0.3759	**c**	0.0623	**d**	0.0564
	e	0.0111						

Exercise D.6.5

1	**a**	0.0228	**b**	0.9332	**c**	0.3085	**d**	0.8849
	e	0.0668	**f**	0.9772				
2	**a**	0.9772	**b**	0.0668	**c**	0.6915	**d**	0.1151
	e	0.9332	**f**	0.0228				
3	**a**	0.3413	**b**	0.1359	**c**	0.0489		
4	**a**	0.6827	**b**	0.1359	**c**	0.3934		
5	**a**	0.8413	**b**	0.4332	**c**	0.7734		
6	**a**	0.1151	**b**	0.1039	**c**	0.1587		
7	**a**	0.1587	**b**	0.6827	**c**	0.1359		
8	**a**	0.1908	**b**	0.4754	**c**	16.88		
9	**a**	0.1434	**b**	0.6595				
10	**a**	0.2425	**b**	0.8413	**c**	0.5050		
11	**a**	−1.2816	**b**	0.2533				
12	**a**	58.2243	**b**	41.7757	**c**	59.80		
13	39.11							
14	9.1660							
15	42%							
16	0.7021							
17	**a**	0.2903	**b**	0.4583	**c**	0.2514		
18	23%							
19	0.5							
20	11%							
21	5%							
22	14%							
23	1.8							
24	252							
25	0.1517							
26	0.3821							
27	0.22							
28	322							
29	0.1545							

30	7								
31	87								
32	**a i**	0.0062	**ii**	0.0478	**iii**	0.9460	**b**	0.0585	
33	**a**	$5.11	**b**	$7.39					
34	**a**	0.0062	**b i**	0.7887	**ii**	0.0324	**c**	$1472	

35. Given that $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, where $a < 0$.

37. : $\mu = 31.5$ and $\sigma = 22.0$.

38. $Q_1 = 6.62$ and $Q_3 = 13.38$

39. $\sigma = 1.48$. and $Q_1 = 9$.

40. Mathematics

41. $\mu = 5.83$ and $\sigma = 2.410$.

42. binomial 0.0551, normal 0.0807

43	**a**	$\mu = 66.86$, $\sigma = 10.25$		**b**	$0.38*S*			
44	**a**	$\mu = 37.2$, $\sigma = 28.2$		**b**	20 (19.9)			
45	**a i**	0.3446	**ii**	0.2347	**b i**	0.3339	**ii**	0.3852
	c	0.9995						

Exercise D.6.2

16 A game is played by selecting coloured discs from a box. The box initially contains two red and eight blue discs. Tom pays \$10.00 to participate in the game. Each time Tom participates he selects two discs. The winnings are governed by the probability distribution shown below, where the random variable N is the number of red discs selected.

n Winnings	0 \$0	1 \$W	2 \$5W
$P(N = n)$			

a Complete the table.

b For what value of W will the game be fair?

17. A random variable X has the following probability distribution:

x	0	1	2
$P(X = x)$	a	$\frac{1}{3}(1 - b)$	$\frac{1}{3}b$

a What values may a and b take?

b Express, in terms of a and b : **i** $E(X)$ **ii** $Var(X)$.

18. a Find the mean and variance of the probability distribution defined by:

$P(Z = z) = k(0.8)^z, z = 0, 1, 2, \ldots$

bi Show $P(X = x) = p \times (1 - p)^x, x = 0, 1, 2, \ldots$ defines a probability distribution.

ii Show $E(X) = \frac{1 - p}{p}$.

iii Show $Var(X) = \frac{1 - p}{p^2}$.

Exercise D.6.3

16. In a suburb, it is known that 40% of the population are blue-collar workers. A delegation of one hundred volunteers are each asked to sample 10 people in order to determine if they are blue-collar workers. The town has been divided into 100 regions so that there is no possibility of doubling up (i.e. each worker is allocated one region). How many of these volunteers would you expect to report that there were fewer than 4 blue-collar workers?

17. Show that if $X \sim B(n, p)$, then:

$$P(X = x + 1) = \left(\frac{n-x}{x+1}\right)\left(\frac{p}{1-p}\right)P(X = x),\ x = 0, 1, 2, \ldots, n-1$$

18. Show that if $X \sim B(n, p)$, then:

a $E(X) = np$. b $Var(X) = np(1-p)$.

19. Mifumi has ten pots labelled one to ten. Each pot and its content can be considered to be identical in every way. Mifumi plants a seed in each pot, such that each seed has a germinating probability of 0.8.

a Find the probability that:

i all the seeds will germinate.

ii exactly three seeds will germinate.

iii more than eight seeds germinate.

b How many pots must Mifumi use to be 99.99% sure to obtain at least one flower?

20. A fair die is rolled eight times. If the random variable X denotes the number of fives observed, find:

a $E(X)$. **b** $Var(X)$. **c** $E\left(\frac{1}{8}X\right)$. **d** $Var\left(\frac{1}{8}X\right)$.

21. A bag contains 5 balls of which 2 are red. A ball is selected at random. Its colour is noted and then it is replaced in the bag. This process is carried out 50 times. Find:

a the mean number of red balls selected.

b the standard deviation of the number of red balls selected.

22. The random variable X is $B(n, p)$ distributed such that $\mu = 9$ and $\sigma^2 = 3.6$. Find:

a $E(X^2 + 2X)$. **b** $P(X = 2)$.

23. a If $X \sim Bin(10, 0.6)$, find: **i** $E(X)$. **ii** $Var(X)$.

b If $X \sim Bin(15, 0.4)$, find: **i** $E(X)$. **ii** $Var(X)$.

24. The random variable X has a binomial distribution such that $E(X) = 12$ and $Var(X) = 4.8$. Find $P(X = 12)$.

25. Metallic parts produced by an automated machine have some variation in their size. If the size exceeds a set threshold, the part is labelled as defective. The probability that a part is defective is 0.08. A random sample of 20 parts is taken from the day's production. If X denotes the number of defective parts in the sample, find its mean and variance.

26. Quality control for the manufacturing of bolts is carried out by taking a random sample of 15 bolts from a batch of 10,000. Empirical data shows that 10% of bolts are found to be defective. If three or more defectives are found in the sample, that particular batch is rejected.

a Find the probability that a batch is rejected.

b The cost to process the batch of 10,000 bolts is $20.00. Each batch is then sold for $38.00, or it is sold as scrap for $5.00 if the batch is rejected. Find the expected profit per batch.

27. In a shooting competition, a competitor knows (that on average) she will hit the bulls-eye on three out of every five attempts. If the competitor hits the bulls-eye she receives $10.00.

However, if the competitor misses the bulls-eye but still hits the target region she only receives $5.00.

a What can the competitor expect in winnings on any one attempt at the target?

b How much can the competitor expect to win after 20 attempts?

28. A company manufactures bolts which are packed in batches of 10,000. The manufacturer operates a simple sampling scheme whereby a random sample of 10 is taken from each batch. If the manufacturer finds that there are fewer than 3 faulty bolts the batch is allowed to be shipped out. Otherwise, the whole batch is rejected and reprocessed.

a If 10% of all bolts produced are known to be defective, find the proportion of batches that will be reprocessed.

b Show that if $100p\%$ of bolts are known to be defective, then P(Batch is accepted) = $(1-p)^8(1+8p+36p^2)$, $0 \le p \le 1$

c Using a graphics calculator, sketch the graph of P('Batch is accepted') versus p.

Describe the behaviour of this curve.

29. Large batches of screws are produced by TWIST'N'TURN Manufacturers Ltd. Each batch consists of N screws and has a proportion p of defectives. It is decided to carry out an inspection of the product, by selecting 4 screws at random and accepting the batch if there is no more than one defective, otherwise the batch is rejected.

a Show that P(Accepting any batch) = $(1-p)^3(1+3p)$.

b Sketch a graph showing the relationship between the probability of accepting a batch and p (the proportion of defectives).

30. A quality control process for a particular electrical item is set up as follows:

A random sample of 20 items is selected. If there is no more than one faulty item the whole batch is accepted. If there are more than two faulty items the batch is rejected. If there are exactly two faulty items, a second sample of 20 items is selected from the same batch and is accepted only if this second sample contains no defective items.

Let p be the proportion of defectives in a batch.

a Show that the probability, $\Phi(p)$, that a batch is accepted is given by:

$$\Phi(p) = (1-p)^{19}[1+19p+190p^2(1-p)^{19}],\ 0 \le p \le 1.$$

b Find the probability of accepting this batch if it is known that 5% of all items are defective.

c If 200 such batches are produced each day, find an estimate of the number of batches that can be expected to be rejected on any one day.

Challenging question!

31. Given that the random variable X denotes the number of successes in n Bernoulli trials, with probability of success on any given trial represented by p:

a find $E(X|X \geq 2)$. **b** show that $\sigma \leq \frac{1}{2}\sqrt{n}$.

Exercise D.6.5

1. If Z is a standard normal random variable, find:

   c $p(Z \geq 0.5)$ d $p(Z \leq 1.2)$ e $p(Z \geq 1.5)$ f $p(Z \leq 2)$

2. If Z is a standard normal random variable, find:

   c $p(Z \geq -0.5)$ d $p(Z \leq -1.2)$ e $p(Z \geq -1.5)$ f $p(Z \leq -2)$

3. If Z is a standard normal random variable, find:

   c $p(1.5 \leq Z < 2.1)$

4. If Z is a standard normal random variable, find:

   c $p(-1.5 \leq Z < -0.1)$

5. If X is a normal random variable with mean $\mu = 8$ and variance $\sigma^2 = 4$, find:

   c $p(X < 9.5)$

6. If X is a normal random variable with mean $\mu = 100$ and variance $\sigma^2 = 25$, find:

   c $p(X < 95)$

7. If X is a normal random variable with mean $\mu = 60$ and standard deviation $\sigma = 5$, find:

   c $p(50 \leq X < 55)$

17. For a normal variable, X, $\mu = 196$ and $\sigma = 4.2$. Find:

   c $p(193.68 < X < 196.44)$

43.

   a Find the mean and standard deviation of the normal random variable X, given that $P(X < 50) = 0.05$ and $P(X > 80) = 0.1$.

   b Electrical components are mass-produced and have a measure of 'durability' that is normally distributed with mean μ and standard deviation 0.5.

   The value of μ can be adjusted at the control room. If the measure of durability of an item scores less than 5, it is classified as defective. Revenue from sales of non-defective items is \$ S per item, while revenue from defective items is set at $\$^1/_{10}S$. Production cost for these components is set at $\$^1/_{10}\mu S$. What is the expected profit per item when μ is set at 6?

44. From one hundred first year students sitting the end-of-year Botanical Studies 101 exam, 46 of them passed while 9 were awarded a high distinction.

   a Assuming that the students' scores were normally distributed, determine the mean and variance on this exam if the pass mark was 40 and the minimum score for a high distinction was 75.

   Some of the students who failed this exam were allowed to sit a 'make-up' exam in early January of the following year. Of those who failed, the top 50% were allowed to sit the 'make-up' exam.

   b What is the lowest possible score that a student can be awarded in order to qualify for the 'make-up' exam.

3745deviation of 5 cm. A man is selected at random from this population.

a Find the probability that this person is:

i at least 180 cm tall

ii between 177 cm and 180 cm tall.

b Given that the person is at least 180 cm, find the probability that he is:

i at least 184 cm

ii no taller than 182 cm.

c If ten such men are randomly selected, what are the chances that at least two of them are at least 176 cm?

Exercise D.6.1

1

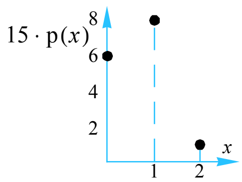
x	0	1	2	3	4
$P(x)$	0	0.1	0.2	0.5	0.2

3 0.3

4 **a** 0.1 **bi** 0.2 **ii** 0.7

5 **a** $p(0) = \frac{6}{15}, p(1) = \frac{8}{15}, p(2) = \frac{1}{15}$

b



c $\frac{14}{15}$

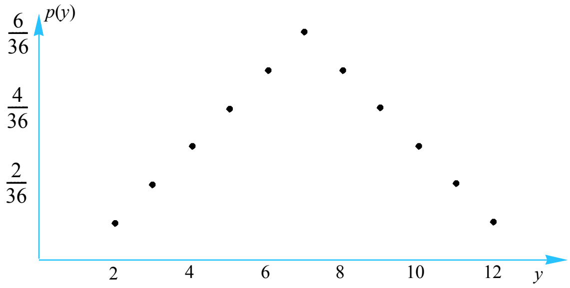
6 **a** $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

b

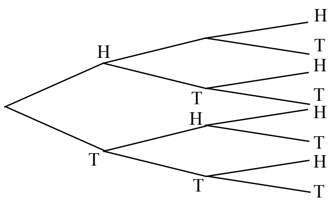
x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

c $\frac{5}{36}$

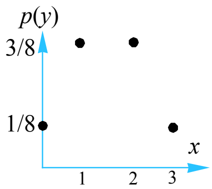
d



7 **a**



b



$p(0) = \frac{1}{8}$

$p(1) = \frac{3}{8}$

$p(2) = \frac{3}{8}$

$p(3) = \frac{1}{8}$

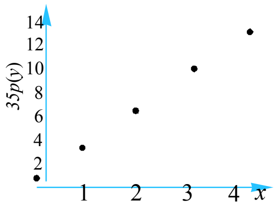
c $\frac{4}{7}$

8 **a** $\frac{1}{35}$ **bi** $p(0) = \frac{1}{35}$ $p(1) = \frac{4}{35}$

$p(2) = \frac{7}{35}$ $p(3) = \frac{10}{35}$

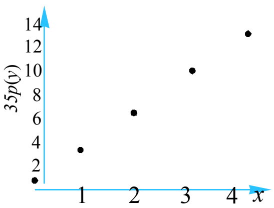
$p(4) = \frac{13}{35}$

ii



c $\frac{6}{7}$

9



a i 0.9048 **ii** 0.09048 **b** 0.0002

10 0.3712

11 **a** $p(0) = \frac{11}{30}, p(-1) = \frac{1}{2}, p(3) = \frac{2}{15}$ **b i** $\frac{11}{30}$ **ii** $\frac{13}{15}$

12

n	0	1	2
P(N = n)	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

13 **a**

n	1	2	3	4
$P(N = n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b

s	2	3	4	5	6	7	8
P(S = s)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

14 **a** 0.81 **b** 0.2439

Exercise D.6.2

1 **a** 2.8 **b** 1.86

2 **a** 3 **b i** 1 **ii** 1 **c i** 6 **ii** 0.4

3 **a i** 1.3 **ii** 2.5 **iii** –0.1

b i 0.9 **ii** 7.29 **c i** $\frac{31}{60}$ **ii** 0.3222

4 $\mu = \frac{2}{3}, \sigma^2 = 0.3556$

5 **a** 7 **b** 5.8333

6 $np = 3 \times \frac{1}{2} = 1.5$

7 **a** $\frac{1}{25}$ **b** 2.8 **c** 1.166

8 **a** 0.1 **b i** 0.3 **ii** 1

c i 0 **ii** 1 **iii** 2

9 5.56

10 $p(0) = \frac{35}{120}, p(1) = \frac{63}{120}, p(2) = \frac{21}{120}, p(3) = \frac{1}{120}$ **b i** 0.9 **ii** 0.49

c $W = 3N - 3, E(W) = -0.3$

11 **a** \$ –1.00 **b** both the same

12 **a** 50 **b** 18 **c** 2

13 **a** 11 **b** $\frac{\sqrt{3}}{3}$ **c** –4

14 **a** 0.75 **b** 0.6339

15 **a** $E(X) = 1 - 2p$, $Var(X) = 4p(1-p)$ **b i** $n(1-2p)$ **ii** $4np(1-p)$

16 **a**

n	0	1	2
P(N = n)	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

b $W = 21.43$

17 **a** $a = \frac{2}{3}, 0 \le b \le 1$ **b** $E(X) = \frac{b+1}{3}$, $Var(X) = \frac{1}{9}(2 + 7b - b^2)$

18 **a** $E(X) = 4$, $Var(X) = 20$

Exercise D.6.3

1 **a** 0.2322 **b** 0.1737 **c** 0.5941

2 **a** 0.3292 **b** 0.8683 **c** 0.2099 **d** 0.1317

3 **a** 0.1526 **b** 0.4812 **c** 0.5678

4 **a** 0.7738 **b** 3.125×10^{-7} **c** 0.9988 **d** 3×10^{-5}

5 **a** 0.2787 **b** 0.4059

6 **a** 0.2610 **b** 0.9923

7 **a** 0.2786 **b** 0.7064 **c** 0.1061

8 **a** 0.1318 **b** 0.8484 **c** 0.054 **d** 0.326

9 **a** 0.238 **b** 0.6531 **c** 0.0027 **d** 0.726

e 12.86

10 **a** 0.003 **b** 0.2734 **c** 0.6367 **d** 0.648

11 **a** 0.3125 **b** 0.0156 **c** 0.3438 **d** 3

12 **a** 0.2785 **b** 0.3417 **c** 120

13 **a** 0.0331 **b** 0.565

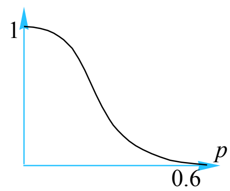
14 **a** 0.4305 **b** 0.61 **c** \$720 **d** 0.2059

15 **a i** 1.4 **ii** 1 **iii** 1.058 **iv** 0.0795

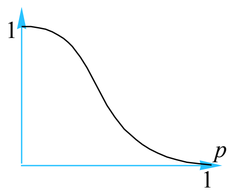
v 0.0047

	b i	3.04	**ii**	3	**iii**	1.373	**iv**	0.2670
	v	0.1390						
16	38.23							
19	**a i**	0.1074	**ii**	7.9×10^{-4}	**iii**	0.3758	**b**	at least 6
20	**a**	$\frac{4}{3}$	**b**	$\frac{10}{9}$	**c**	$\frac{1}{6}$	**d**	$\frac{5}{288}$
21	**a**	20	**b**	3.4641				
22	**a**	102.6	**b**	0.000254				
23	**a i**	6	**ii**	2.4	**b i**	6	**ii**	3.6
24	0.1797							
25	1.6, 1.472							
26	**a** 0.1841 **b** $11.93							
27	**a** $8 **b** $160							

28 **a** 0.0702 **c**



29 **b**



30 **b** 0.8035 **c** 39.3

31 **a** $\dfrac{np - np(1-p)^{n-1}}{1-(1-p)^n - np(1-p)^{n-1}},\ 0 < p < 1$

Exercise D.6.4

1	**a**	0.6915	**b**	0.9671	**c**	0.9474	**d**	0.9965
	e	0.9756	**f**	0.0054				
2	**a**	0.0360	**b**	0.3759	**c**	0.0623	**d**	0.0564
	e	0.0111						

Exercise D.6.5

1	**a**	0.0228	**b**	0.9332	**c**	0.3085	**d**	0.8849
	e	0.0668	**f**	0.9772				
2	**a**	0.9772	**b**	0.0668	**c**	0.6915	**d**	0.1151
	e	0.9332	**f**	0.0228				
3	**a**	0.3413	**b**	0.1359	**c**	0.0489		
4	**a**	0.6827	**b**	0.1359	**c**	0.3934		
5	**a**	0.8413	**b**	0.4332	**c**	0.7734		
6	**a**	0.1151	**b**	0.1039	**c**	0.1587		
7	**a**	0.1587	**b**	0.6827	**c**	0.1359		
8	**a**	0.1908	**b**	0.4754	**c**	16.88		
9	**a**	0.1434	**b**	0.6595				
10	**a**	0.2425	**b**	0.8413	**c**	0.5050		
11	**a**	−1.2816	**b**	0.2533				
12	**a**	58.2243	**b**	41.7757	**c**	59.80		
13	39.11							
14	9.1660							
15	42%							
16	0.7021							
17	**a**	0.2903	**b**	0.4583	**c**	0.2514		
18	23%							
19	0.5							
20	11%							
21	5%							
22	14%							
23	1.8							
24	252							
25	0.1517							
26	0.3821							
27	0.22							
28	322							
29	0.1545							

30	7							
31	87							
32	**a i**	0.0062	**ii**	0.0478	**iii**	0.9460	**b**	0.0585
33	**a**	\$5.11	**b**	\$7.39				
34	**a**	0.0062	**b i**	0.7887	**ii**	0.0324	**c**	\$1472

35. Given that $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, where $a < 0$.

37. : $\mu = 31.5$ and $\sigma = 22.0$.

38. $Q_1 = 6.62$ and $Q_3 = 13.38$

39. $\sigma = 1.48$. and $Q_1 = 9$.

40. Mathematics

41. $\mu = 5.83$ and $\sigma = 2.410$.

42. binomial 0.0551, normal 0.0807

43	**a**	$\mu = 66.86, \sigma = 10.25$			**b**	\$0.38*S*		
44	**a**	$\mu = 37.2, \sigma = 28.2$			**b**	20 (19.9)		
45	**a i**	0.3446	**ii**	0.2347	**b i**	0.3339	**ii**	0.3852
	c	0.9995						

Exercise D.12.1

13. Faults occur randomly along the length of a yarn of wool where the number of faults per bobbin holding a fixed length of yarn may be assumed to follow a Poisson distribution. A bobbin is rejected if it contains at least one fault. It is known that in the long run 33% of bobbins are rejected.

a Find the probability that a rejected bobbin contains only one fault.

The production manager believes that by doubling the length of yarn on each bobbin there will be a smaller rejection rate. Assuming that the manufacturing process has not altered, is the production manager correct?

Provide a quantitative argument.

14. On average, it is found that 8 out of every 10 electric components produced from a large batch have at least one defective component. Find the probability that there will be at least 2 defective components from a randomly selected batch.

15. Flaws, called seeds, in a particular type of glass sheet occur at a rate of 0.05 per square metre. Find the probability that a rectangular glass sheet measuring 4 metres by 5 metres contains:

a no seeds.

b at least two seeds.

Sheets containing at least two seeds are rejected.

c Find the probability that, in a batch of ten such glass sheets, at most one is rejected.

16. Simar has decided to set up a small business venture. The venture requires Simar to go fishing every Sunday so he can sell his catch on the Monday. He realises that on a proportion p of these days he does not catch anything.

a Find the probability that on any given Sunday, Simar catches:

i no fish. ii one fish. iii at least two fish.

The cost to Simar on any given Sunday if he catches no fish is \$5. If he catches one fish Simar makes a profit of \$2 and if he catches more than one fish he makes a profit of \$10. Let the random variable X denote the profit Simar makes on any given Sunday.

b Show that $E(X) = 10 - 15p + 8p\ln p$, $0 < p < 1$

c Find the maximum value of p, if Simar is to make a positive gain on his venture.

Exercise D.12.1

1 a	$P(X=x)=\frac{e^{-2}.2^x}{x!}, x=0,1,2,...$							
	b	i	0.1353	ii	0.2707	iii	0.5940	iv 0.4557
2	a	0.0383	b	0.1954				
3	a	0.2052	b	0.9179				
4	a	0.2623	b	0.8454				
5	a	0.0265	b	0.0007				
6	a	0.1889	b	0.7127				
7	a	0.7981	b	0.2019	c	0.1835		
8	a	0.2661	b	0.5221				
9	0.1912							
10	a	0.3504	b	0.6817				
11	a	0.00127	b	0.0500				
12	a	0.1804	b	0.0166	c	0.3233		
13	a	0.8131; 0.5511		No				
14	14. 0.4781							
15	a	0.3679	b	0.2642	c	0.2135		
16	a	i p	ii	$-p\ln p$	iii	$-p+p\ln p$	c	0.4785

Example

The concentration of a drug, in milligrams per millilitre, in a patient's bloodstream, t hours after an injection, is approximately modelled by the function:

$$t \mapsto \frac{2t}{8+t^2}, t \geq 0$$

Find the average rate of change in the concentration of the drug present in a patient's bloodstream:

a during the first hour

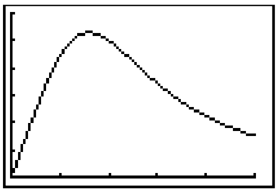
b during the first two hours

c during the period $t = 2$ to $t = 4$.

To help us visualise the behaviour of this function we will make use of the TI–83.

Begin by introducing the variable C, to denote the concentration of the drug in the patient's bloodstream t hours after it is administered.

So that $C(t) = \frac{2t}{8+t^3}, t \geq 0$.



Initially the concentration is 0 milligrams per millilitre. The concentration after 1 hour is given by $C(1) = \frac{2 \times 1}{8+1^3} = \frac{2}{9} \approx 0.22$.

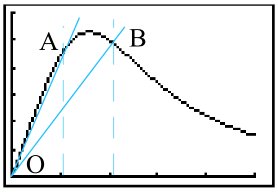
Therefore, the average rate of change in concentration (C_{ave}) during the first hour is given by $C_{ave} = \frac{0.22-0}{1-0} = 0.22$. Note: the units are *mg/mL/hr*.

The concentration 2 hours after the drug has been administered is $C(2) = \frac{2 \times 2}{8+2^3} = 0.25$. That is, 0.25 *mg/ml.*

Therefore, the average rate of change in concentration with respect to time is: $C_{ave} = \frac{0.25-0}{2-0} = 0.125$.

Notice that although the concentration has increased (compared to the concentration after 1 hour), the rate of change in the concentration has actually decreased!

This should be evident from the graph of $C(t)$ versus t.



The slope of the straight line from the origin to A(1, 0.22), m_{OA}, is greater than the slope from the origin O to the point B(2, 0.25), m_{OB}.

That is $m_{OA} > m_{OB}$.

The average rate of change in concentration from $t = 2$ to $t = 4$ is given by $\frac{C(4) - C(2)}{4 - 2}$.

Now, $\frac{C(4) - C(2)}{4 - 2} = \frac{\frac{2 \times 4}{8 + 4^3} - 0.250}{4 - 2} \approx \frac{0.111 - 0.250}{2} = -0.0694$

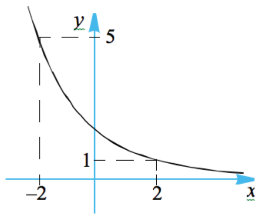
Therefore, the average rate of change of concentration is -0.070 *mg/ml/hr*,

i.e. the overall amount of drug in the patient's bloodstream is decreasing during the time interval $2 \le t \le 4$.

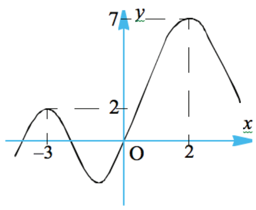
Exercise E.1.2

1. For each of the following graphs determine the average rate of change over the specified domain.

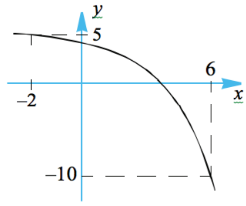
c $x \in [-2, 2]$



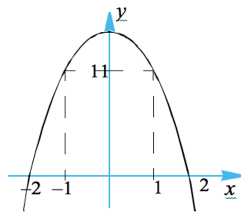
d $x \in [-3, 2]$



e $x \in [-2, 6]$



f $x \in [-1, 1]$



8. For the case where r is 20 cm,

   a find the average rate of increase in the amount of water inside the bowl with respect to its height, h cm, as the water level rises from 2 cm to 5 cm.

   b Find the average rate of increase in the amount of water inside the bowl with respect to its height, h cm, as the water level rises by

   i 1 cm ii 0.1 cm iii 0.01 cm.

9. An amount of money is placed in a bank and is accumulating interest on a daily basis. The table below shows the amount of money in the savings account over a period of 600 days.

t (days)	100	200	300	400	500	600	700
$\$D$/day	1600	1709	1823	1942	2065	2194	2328

   a Plot the graph of $\$D$ versus t (days).

   b Find the average rate of change in the amount in the account during the period of 100 days to 300 days.

10. The temperature of coffee since it was poured into a cup was recorded and tabulated below.

t min	0	2	4	6	9
T °C	60	50	30	10	5

a Plot these points on a set of axes that show the relationship between the temperature of the coffee and the time it has been left in the cup.

b Find the average rate of change of temperature of the coffee over the first 4 minutes.

c Over what period of time is the coffee cooling the most rapidly?

11. The displacement, d metres, of an object, t seconds after it was set in motion is described by the equation:

$$d = 4t + 5t^2 \text{, where } t \geq 0.$$

a Find the distance that the object travels in the first 2 seconds of its motion.

b Find the average rate of change of distance with respect to time undergone by the object over the first 2 seconds of its motion.

c What quantity is being measured when determining the average rate of change of distance with respect to time?

d How far does the object travel during the 5th second of motion?

e Find the object's average speed during the 5th second.

12. A person invests $1000 and estimates that, on average, the investment will increase each year by 16% of its value at the beginning of the year.

a Calculate the value of the investment at the end of each of the first 5 years.

b Find the average rate at which the investment has grown over the first 5 years.

Exercise E.1.3

5. For each of the functions, f, given below, find the gradient of the secant joining the points $P(a, f(a))$ and $Q(a+h, f(a+h))$ and hence deduce the gradient of the tangent drawn at the point P.

   a $f(x) = x$ b $f(x) = x^2$

   c $f(x) = x^3$ d $f(x) = x^4$.

   Hence deduce the gradient of the tangent drawn at the point $P(a, f(a))$ for the function $f(x) = x^n, n \in N$.

6. The healing process of a certain type of wound is measured by the decrease in surface area that the wound occupies on the skin. A certain skin wound has its surface area modelled by the equation $S(t) = 20 \times 2^{-0.1t}$ where S sq. cm is the unhealed area t days after the skin received the wound.

   a Sketch the graph of $S(t) = 20 \times 2^{-0.1t}$, $t \geq 0$.

   b i What area did the wound originally cover?

   ii What area will the wound occupy after 2 days?

   iii How much of the wound healed over the two day period?

   iv Find the average rate at which the wound heals over the first two days.

   c How much of the wound would heal over a period of h days?

   d Find the rate at which the wound heals:

   i immediately after it occurs

   ii one day after it occurred.

Exercise E.1.1

1. 4
2. 2
3. 4
4. 6
5. 0
6. i $^5/_9$ ii $^7/_9$ iii $^3/_7$ iv $^5/_{11}$
7. This is a tricky one and still a matter for debate. If you approach zero from the positive side, you approach 1. You cannot approach from the negative side.
8. ~2.71828... This is *e* - Euler's number and very important!

Exercise E.1.2

1 **a** $\frac{3}{4}$ **b** $\frac{3a}{4b}$ **c** –1 **d** 1
e $-\frac{15}{8}$ **f** 0

2 **a** 4 **b** 0.2 **c** 0.027 **d** 0.433
e –0.01 **f** 6.34 **g** 6.2 **h** 0

3 **a** 6 m/s **b** 30 m/s **c** $11 + 6h + h^2$ m/s

4 12 m/s

5 $8 + 2h$

6 –3.49°C/sec

7 **a** 127π cm³/cm
b i 19.6667π cm³/cm **ii** 1.9967π cm³/cm **iii** 0.2000π cm³/cm

8 1.115

9 **a** –7.5°C/min **b** $t = 2$ to $t = 6$

10 **a** 28 m **b** 14 m/s **c** average speed
d 49 m **e** 49 m/s

11 **a** \$1160, \$1345.6, \$1560.90, \$1810.64, \$2100.34 **b** \$220.07 per year

Exercise E.1.3

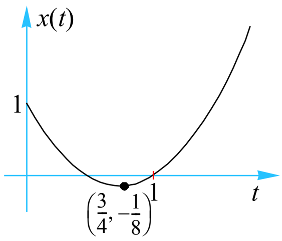
1 **a** $h + 2$ **b** $4 + h$ **c** $\frac{-1}{1+h}$ **d** $3 - 3h + h^2$

2 **a** 2 **b** 4 **c** –1 **d** 3

3 **a** $2a + h$ **b** $-(2a + h)$ **c** $(2a + 2) + h$ **d** $3a^2 + 1 + 3ah + h^2$
e $-(3a^2 + 3ah + h^2)$ **f** $3a^2 - 2a + (3a - 1)h + h^2$

g $\frac{-2}{a(a+h)}$ **h** $-\frac{1}{(a-1)(a-1+h)}$ **i** $\frac{1}{\sqrt{a+h}+\sqrt{a}}$

4 **a**

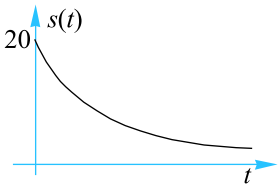


b i 3 ms^{-1} **ii** 2 ms^{-1} **iii** 1.2 ms^{-1}

d Find (limit) as $h \to 0$ **e** 4t – 3

5 **a** 1 ; 1 **b** $2a + h$; $2a$ **c** $3a^2 + 3ah + h^2$; $3a^2$ **d** $4a^3 + 6a^2h + 4ah^2 + h^3$; $4a^3$

6 **a**



b i 20 cm^2 **ii** 17.41 cm^2 **iii** 2.59

Exercise E.1.4

1 **a** 3 **b** 8 **c** $-\frac{1}{9}$ **d** 1.39

e –1 **f** $\frac{17}{16}$

2 **a** 4.9 m **b** $4.9(h^2 + 2h)$ m **c** 9.8 m/s

3 **a** $8x$ **b** $10x$ **c** $12x^2$ **d** $15x^2$

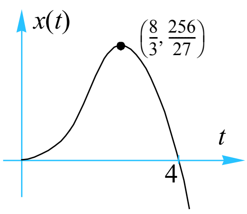
e $16x^3$ **f** $20x^3$

4 **a** $4x$ **b** –1 **c** $-1 + 3x^2$ **d** $-x^{-2}$

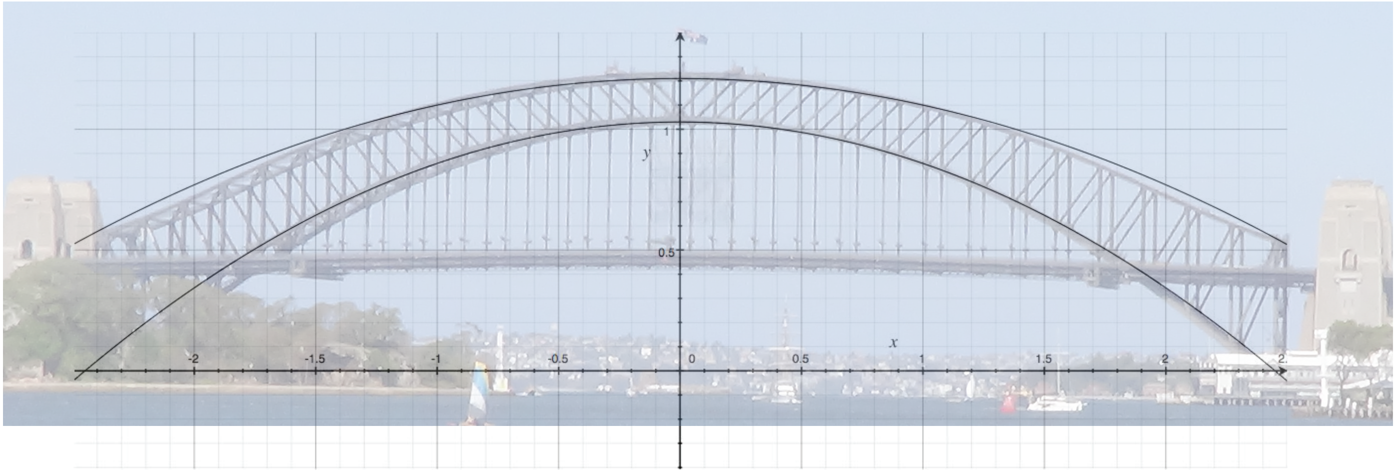
e $-2(x+1)^{-2}$ **f** $0.5x^{-1/2}$

5 **a** 1 ms^{-1} **b** $(2 - a)$ ms^{-1}

6 **a**



b i 5 ms^{-1} **ii** 4 ms^{-1} **c** $8t - 3t^2$ ms^{-1} **d** $\frac{8}{3}$ sec


y
x
1
0.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Gradient Finder

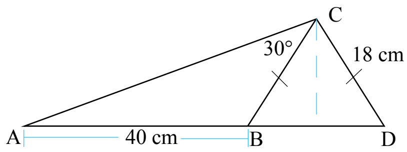
Mid-point	1
Increment	0.01

x	f(x)	Difference	Approx Gradient
0.97	0.97174771		
0.98	0.98078434	0.00903663	0.90366328
0.99	0.99019802	0.00941368	0.94136811
1	1	0.00980198	0.98019785
1.01	1.01020202	0.01020202	1.02020219
1.02	1.02081635	0.01061433	1.06143313
1.03	1.0318558	0.01103945	1.10394517

Exercise C.2.4

8. The framework for an experimental design for a kite is shown. Material for the kite costs $12 per square cm.

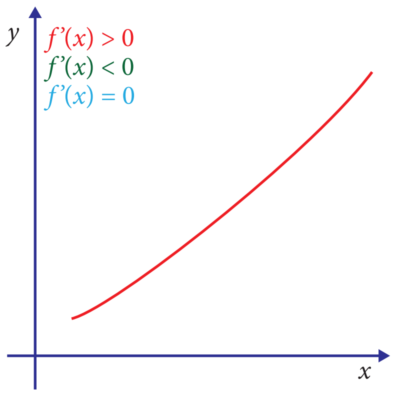
How much will it cost for the material if it is to cover the framework of the kite.



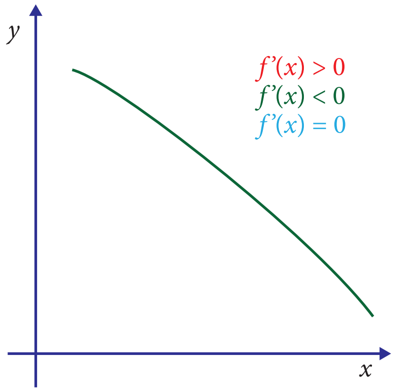
9. A boy walking along a straight road notices the top of a tower at a bearing of 284°T. After walking a further 1.5 km he notices that the top of the tower is at a bearing of 293°T. How far from the road is the tower?

Exercise E.2.1

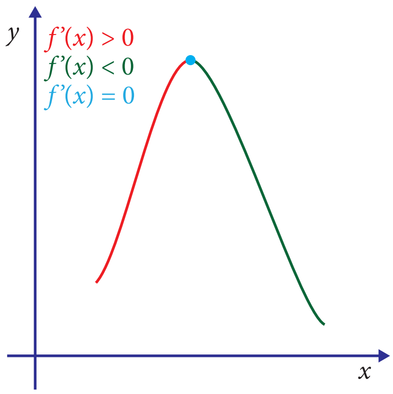
1. a



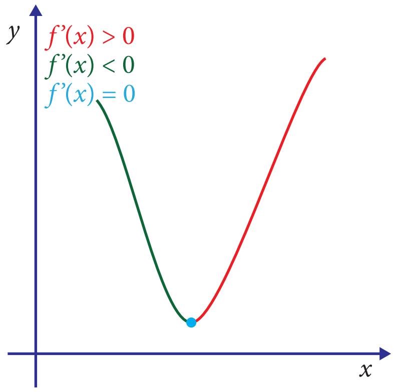
b



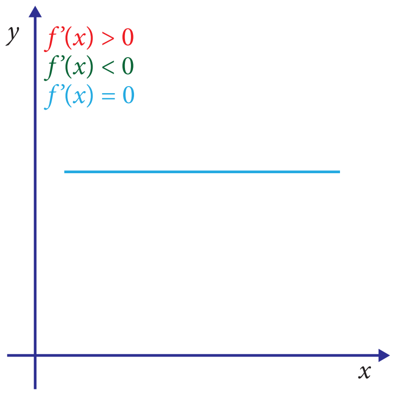
c



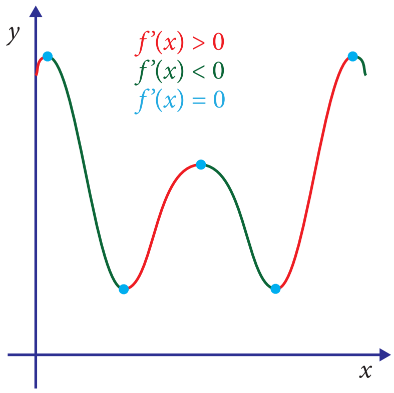
d



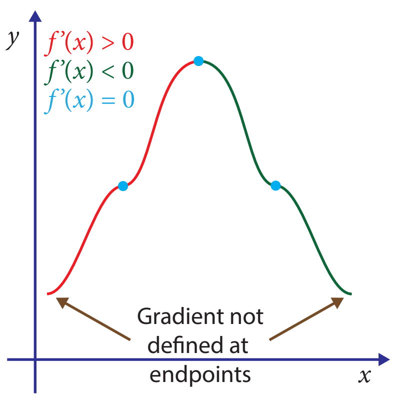
e



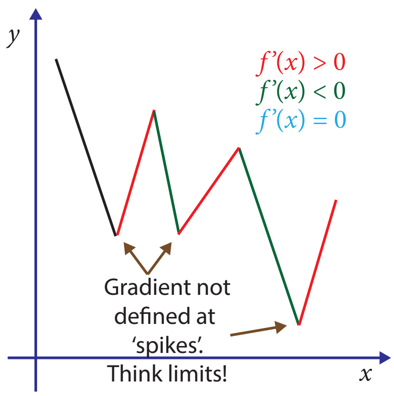
f



g



h



Exercise E.2.2

1 **a** $5x^4$ **b** $9x^8$ **c** $25x^{24}$ **d** $27x^2$

e $-28x^6$ **f** $2x^7$ **g** $2x$ **h** $20x^3 + 2$

i $-15x^4 + 18x^2 - 1$ **j** $-\frac{4}{3}x^3 + 10$

k $9x^2 - 12x$ **l** $3 + \frac{2}{5}x + 4x^3$

2 **a** $8x - 4$ **b** $-\frac{1}{x^2} - 2$ **c** $2x - 6$ **d** $\frac{-4}{x^5}$

e $\frac{-2x-2}{x^3}$ **f** $2x - 1$ **g** $3x^2 - 4x + 3$ **h** $4x^3 - 8x$

i $3x^2 - \frac{3}{x^2} + \frac{3}{x^4} - 3$

4 **a** $6x^2$ **b** $3x^2 + 1$ **c** $4x^3 - 3x^2$ **d** $2x$

e $2x + 1$ **f** $3x^2 + 6x + 1$ **g** $4x^3 + 3x^2 + 2x$ **h** $3x^2 + 6x$

Exercise E.2.3

1 **a** $48t^3$ **b** $2n - \frac{2}{n^2} - \frac{4}{n^5}$ **c** $\frac{12}{r^2} - \frac{18}{r^3}$

d $3\theta^2 - \frac{3}{\theta^2} - \frac{3}{\theta^4} + 3$ **e** $40 - 3L^2$ **f** $-\frac{100}{v^3} - 1$

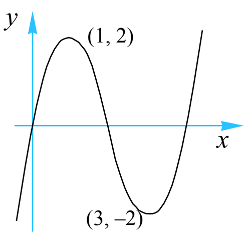
g $6l^2 + 5$ **h** $2\pi + 8h$

2 **a** $\frac{8}{3t^3}$ **b** $2\pi r - \frac{20}{r^2}$ **c** $4s^3 + \frac{3}{s^2}$

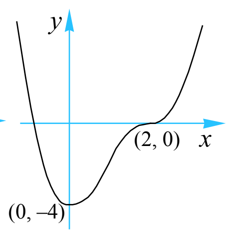
d $\frac{-1}{t^2} + \frac{2}{t^3} - \frac{6}{t^4}$ **e** $1 - \frac{16}{b^2}$ **f** $3m^2 - 4m - 4$

Exercise E.2.4

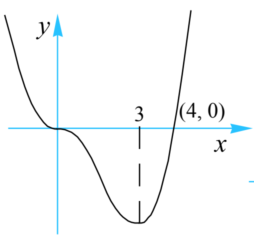
1 **a**



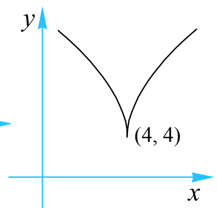
b



c



d



2 **a** max at (1, 4) **b** min at $\left(-\frac{9}{2}, -\frac{81}{4}\right)$ **c** min at (3, –45) max (–3, 63)

d max at (0, 8), min at (4, -24) **e** max at (1, 8), min at (–3, –24)

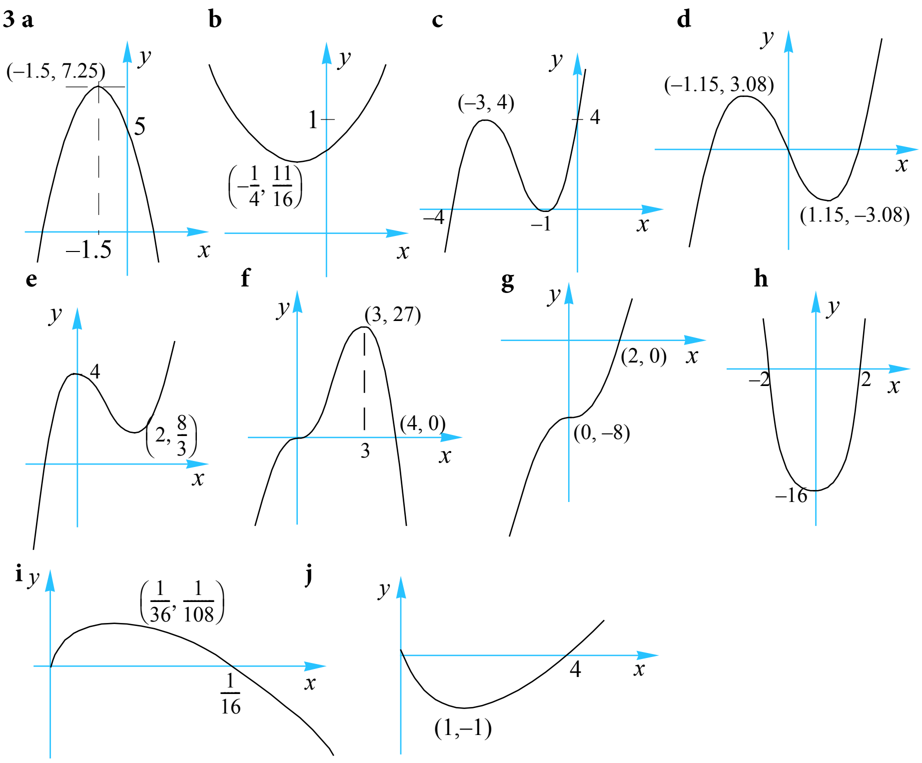
f min at $\left(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27}\right)$, max at $\left(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27}\right)$ **g** min at (1, –1)

h max at (0, 16), min at (2, 0), min at (–2, 0) **i** min at (1, 0) max at $\left(-\frac{1}{3}, \frac{32}{27}\right)$

j min at $\left(\frac{4}{9}, -\frac{4}{27}\right)$ **k** min at (2, 4), max at (–2, –4)

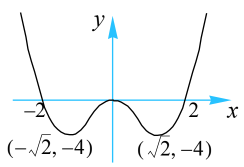
l min at (1, 2), min at (–1, 2)

3



4 min at (1, –3), max at (–3, 29), non-stationary infl (–1, 13)

5



Exercise E.3.1

Approximate answers only. Tangent given first.

1.	a	1,−1	b	−1,1	c	$3,-{}^{1}/_{3}$
2.	a	−2,½	b	−1,1	c	−½,2
3.	a	2,−½	b	undefined	c	½,−2

Exercise E.3.2

1	**a**	$y = 7x - 10$	**b**	$y = -4x + 4$	**c**	$4y = x + 5$
	d	$y = {}^{1}/_{6}x + {}^{3}/_{2}$	**e**	$y = -2x + 5$	**f**	$y = 2$
	g	$y = -4x - 3$	**h**	$y = 12x - 4$	**i**	$y = {}^{15}/_{4}x - 1{}^{1}/_{4}$
2	**a**	$7y = -x + 30$	**b**	$4y = x - 1$	**c**	$y = -4x + 14$
	d	$y = -6x + 57$	**e**	$y = ½x + 2½$	**f**	$x = 1$
	g	$y = ¼x + 1¼$	**h**	$y = -{}^{1}/_{12}x + 8{}^{1}/_{12}$	**i**	$y = -{}^{4}/_{15}x + 6{}^{47}/_{60}$

3 A: $y = 28x - 44$, B: $y = -28x - 44$, Isosceles. $z \equiv (0, a^2 - 3a^4)$

4 2 sq. units, $y = 2$ $x = 1$

5 $y = 4x - 9$

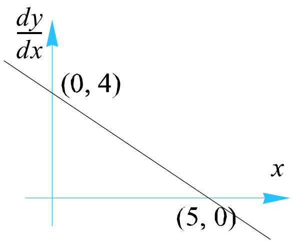
6 A: $y = -8x + 32$, B: $y = 6x + 25$, $\left(\frac{1}{2}, 28\right)$

7 $m = -2$, $n = 5$

Exercise E.6.2

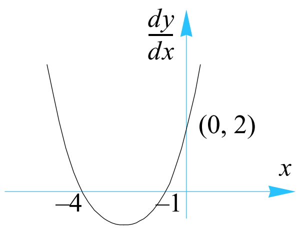
11. Sketch the graph of $y = f(x)$ for each of the following:

a



Where the curve passes through the point (5, 10).

b



Where the curve passes through the point (0, 0).

12. Find $f(x)$ given that $f''(x) = 12x + 4$ and that the gradient at the point (1, 6) is 12.

13. Find $f(x)$ given that $f'(x) = ax^2 + b$, where the gradient at the point (1, 2) is 4, and that the curve passes through the point (3, 4).

Extensions.

14. The rate at which a balloon is expanding is given by

$$\frac{dV}{dt} = kt^{4.5},\ t \geq 0,$$

where t is the time in minutes since the balloon started to be inflated and V cm³ is its volume. Initially the balloon (which may be assumed to be spherical) has a radius of 5 cm. If the balloon has a volume of 800 cm³ after 2 minutes, find its volume after 5 minutes.

15. The area, A cm², of a healing wound caused by a fall on a particular surface decreases at a rate given by the equation:

$$A'(t) = -\frac{35}{\sqrt{t}}$$

where t is the time in days. Find the initial area of such a wound if after one day the area measures 40 cm².

Exercise E.6.1

1	a	$\frac{1}{4}x^4+c$	b	$\frac{1}{8}x^8+c$	c	$\frac{1}{6}x^6+c$	d	$\frac{1}{9}x^9+c$
	e	$\frac{4}{3}x^3+c$	f	$\frac{7}{6}x^6+c$	g	x^9+c	h	$\frac{1}{8}x^4+c$
2	a	$5x+c$	b	$3x+c$	c	$10x+c$	d	$\frac{2}{3}x+c$
	e	$-4x+c$	f	$-6x+c$	g	$-\frac{3}{2}x+c$	h	$-x+c$
3	a	$x-\frac{1}{2}x^2+c$	b	$2x+\frac{1}{3}x^3+c$	c	$\frac{1}{4}x^4-9x+c$	d	$\frac{2}{5}x+\frac{1}{9}x^3+c$
	e	$\frac{3}{2x^2}-\frac{2}{x}+c$	f	$-\frac{4}{x}+\frac{6}{x^2}-\frac{3}{x^3}+c$				
	g	$\frac{1}{3}x^3+x^2+c$	h	x^3-x^2+c	i	$x-\frac{1}{3}x^3+c$		
4	a	$\frac{1}{3}x^3-\frac{1}{2}x^2-6x+c$	b	$\frac{1}{4}x^4-\frac{2}{3}x^3-\frac{3}{2}x^2+c$	c	$\frac{1}{4}(x-3)^4+c$		
	d	$\frac{2}{5}x^5+\frac{1}{2}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+c$	e	$x-\frac{x^3}{3}+c$				
	f	$\frac{x^4}{256}-\frac{3x^3}{16}+\frac{27x^2}{8}-27x+c$						
5	a	$\frac{1}{2}x^2-3x+c$	b	$2u^2+5u+\frac{1}{u}+c$	c	$-\frac{1}{x}-\frac{2}{x^2}-\frac{4}{3x^3}+c$		
	d	$\frac{1}{2}x^2+3x+c$	e	$\frac{1}{2}x^2-4x+c$	f	$\frac{1}{3}t^3+2t-\frac{1}{t}+c$		
6	a	$\frac{4}{7}4\sqrt{x^7}+2\sqrt{x}-5x+c$	b	$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{4}{7}x^{7/2}-\frac{4}{5}x^{5/2}+c$				
	c	$-\frac{1}{2z^2}+\frac{2}{z}+2z^2+z+c$	d	$\frac{1}{2}t^4+t+c$				
	e	$\frac{2}{5}\sqrt{t^5}-2\sqrt{t^3}+c$	f	$\frac{1}{3}u^3+2u^2+4u+c$				

6. $2\sqrt{x^2-1}$

Exercise E.6.1

1 **a** $\frac{1}{4}x^4 + c$ **b** $\frac{1}{8}x^8 + c$ **c** $\frac{1}{6}x^6 + c$ **d** $\frac{1}{9}x^9 + c$

e $\frac{4}{3}x^3 + c$ **f** $\frac{7}{6}x^6 + c$ **g** $x^9 + c$ **h** $\frac{1}{8}x^4 + c$

2 **a** $5x + c$ **b** $3x + c$ **c** $10x + c$ **d** $\frac{2}{3}x + c$

e $-4x + c$ **f** $-6x + c$ **g** $-\frac{3}{2}x + c$ **h** $-x + c$

3 **a** $x - \frac{1}{2}x^2 + c$ **b** $2x + \frac{1}{3}x^3 + c$ **c** $\frac{1}{4}x^4 - 9x + c$ **d** $\frac{2}{5}x + \frac{1}{9}x^3 + c$

e $\frac{1}{3}x^{3/2} + \frac{1}{x} + c$ **f** $x^{5/2} + 4x^2 + c$ **g** $\frac{1}{3}x^3 + x^2 + c$ **h** $x^3 - x^2 + c$

i $x - \frac{1}{3}x^3 + c$

4 **a** $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + c$ **b** $\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + c$ **c** $\frac{1}{4}(x-3)^4 + c$

d $\frac{2}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$ **e** $x + \frac{1}{2}x^2 - \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$

f $\frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + \frac{2}{3}x^{3/2} - 2x + c$

5 **a** $\frac{1}{2}x^2 - 3x + c$ **b** $2u^2 + 5u + \frac{1}{u} + c$ **c** $-\frac{1}{x} - \frac{2}{x^2} - \frac{4}{3x^3} + c$

d $\frac{1}{2}x^2 + 3x + c$ **e** $\frac{1}{2}x^2 - 4x + c$ **f** $\frac{1}{3}t^3 + 2t - \frac{1}{t} + c$

6 $2\sqrt{x^2 - 1}$

Exercise E.6.2

1 **a** $x^2 + x + 3$ **b** $2x - \frac{1}{3}x^3 + 1$ **c** $y = x^2 - 4$

d $\frac{1}{2}x^2 + \frac{1}{x} + 2x - \frac{3}{2}$ **e** $(x+2)^3$ **f** $y = \frac{x^2}{2} - 3x - \frac{1}{x} + 3\frac{1}{2}$

g $\frac{1}{3}x^3 + 1$ **h** $x^4 - x^3 + 2x + 3$

2 $\frac{1}{2}x^2 + \frac{1}{x} + \frac{5}{2}$

3 \$3835.03

4 9.5

5 $\frac{251}{3}\pi$ cm^3

6 350

7 $y = 3x^2 + 4$

8 1, –8

9 $P(x) = 25 - 5x + \frac{1}{3}x^2$

10 $N = \frac{20000}{201}t^{2.01} + 500, t \geq 0$

11 **a** $y = -\frac{2}{5}x^2 + 4x$ **b** $y = \frac{1}{6}x^3 + \frac{5}{4}x^2 + 2x$

12 $y = 2(x^3 + x^2 + x)$

13 $f(x) = -\frac{3}{10}x^3 + \frac{49}{10}x - \frac{13}{5}$

14 Vol ~ 43 202 cm^3

15 110 cm^2

Exercise E.6.3

1 **a** $\frac{15}{2}$ **b** $\frac{76}{3}$ **c** $\frac{5}{36}$ **d** $-\frac{1}{4}$

2 **a** $\frac{35}{24}$ **b** $\frac{7}{6}$ **c** –2 **d** 0

e $\frac{1}{20}$ **f** $-\frac{4}{3}$ **g** undefined **h** $\frac{5}{6}$

i $\frac{20}{3}$ **j** 0 **k** undefined

3 **a** $\frac{31}{5}$ **b** undefined **c** 0 **d** $\frac{11}{5}$

4 **a** $2m-n$ **b** $m+a-b$ **c** $-3n$ **d** $m(2a-b)$ **e** na^2

5 $\frac{\sqrt{5}}{5}-\frac{\sqrt{21}}{21}$

6 $4\sqrt{\sqrt{3}+1}-4\sqrt{\sqrt{2}+1}$

Exercise E.6.4

1 **a** 4 sq.units **b** $\frac{32}{3}$ sq.units **c** 4 sq.units

d 36 sq.units **e** $\frac{5}{6}$ sq.units

3 12 sq. units

4 $4\left(\sqrt{3}-\frac{1}{3}\right)$ sq.units.

5 2 sq.units.

6 $\frac{37}{12}$ sq. units

7 **a** 0.5 sq. units **b** 1 sq. unit **c** $2(\sqrt{6}-\sqrt{2})$ sq. units

8 $\frac{8}{3}$

9 **a** $\frac{9}{2}$ sq. units **b** 3 sq. units

Exercise E.8.1

1. (a) $\lim_{x\to 0}\left(\frac{x+\sin(2x)}{x-\sin(2x)}\right)=\frac{0}{0}$, so we apply L'Hospital's rule:

$$\lim_{x\to 0}\left(\frac{x+\sin(2x)}{x-\sin(2x)}\right)=\lim_{x\to 0}\left(\frac{1+2\cos(2x)}{1-2\cos(2x)}\right)$$

$$=\frac{1+2}{1-2}=\underline{\underline{-3.}}$$

(b) $\lim_{x\to \pi}\left(\frac{x-\pi}{\sin(x)}\right)=\frac{0}{0}$, so we apply L'Hospital's rule:

$$\lim_{x\to \pi}\left(\frac{x-\pi}{\sin(x)}\right)=\lim_{x\to \pi}\left(\frac{1}{\cos(x)}\right)=\frac{1}{1}=\underline{\underline{1.}}$$

(c) $\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(2x)}{\cos(x)}\right)=\frac{0}{0}$, so we apply L'Hospital's rule:

$$\lim_{x\to \frac{\pi}{2}}\left(\frac{\sin(2x)}{\cos(x)}\right)=\lim_{x\to \frac{\pi}{2}}\left(\frac{2\cos(2x)}{-\sin(x)}\right)=\frac{-2}{-1}=\underline{\underline{2.}}$$

2. (a) $\lim_{x\to \infty}\left(\frac{x}{e^{2x}}\right)=\frac{\infty}{\infty}\Rightarrow$ L'Hospital's rule.

$$\lim_{x\to \infty}\left(\frac{x}{e^{2x}}\right)=\lim_{x\to \infty}\left(\frac{1}{2e^{2x}}\right)=\frac{1}{\infty}=\underline{\underline{0.}}$$

(b) $\lim_{x\to\infty}\left(\frac{\ln(x)}{x}\right) = \frac{\infty}{\infty} \Rightarrow$ L'Hospital's rule

$$\lim_{x\to\infty}\left(\frac{\ln(x)}{x}\right) = \lim_{x\to\infty}\left(\frac{1/x}{1}\right) = \lim_{x\to\infty}\left(\frac{1}{x}\right) = \underline{\underline{0.}}$$

(c) $\lim_{x\to\infty}\left(\frac{2x}{x+\ln(x)}\right) = \frac{\infty}{\infty} \Rightarrow$ L'Hospital's rule.

$$\lim_{x\to\infty}\left(\frac{2x}{x+\ln(x)}\right) = \lim_{x\to\infty}\left(\frac{2}{1+1/x}\right) = \underline{\underline{2.}}$$

3. (a) $\lim_{x\to 0}\left(\frac{2x}{x+\sin(x)}\right) = \frac{0}{0} \Rightarrow$ L'Hospital's rule.

$$\lim_{x\to 0}\left(\frac{2x}{x+\sin(x)}\right) = \lim_{x\to 0}\left(\frac{2}{1+\cos(x)}\right)$$

$$= \frac{2}{1+1} = \underline{\underline{1.}}$$

(b) $\lim_{x\to 0}\left(\frac{\cos(x)-1}{x^2}\right) = \frac{0}{0} \Rightarrow$ L'Hospital's rule.

$$\lim_{x\to 0}\left(\frac{\cos(x)-1}{x^2}\right) = \lim_{x\to 0}\left(\frac{-\sin(x)}{2x}\right) = \frac{0}{0}$$

$\Rightarrow$ L'Hospital's rule again.

$$\lim_{x\to 0}\left(\frac{-\sin(x)}{2x}\right) = \lim_{x\to 0}\left(\frac{-\cos(x)}{2}\right)$$

$$= \underline{\underline{-\frac{1}{2}}}.$$

(e) $\lim_{x \to 0} \left(\frac{x - \sin(x)}{x^3}\right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \left(\frac{1 - \cos(x)}{3x^2}\right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \left(\frac{\sin(x)}{6x}\right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \left(\frac{\cos(x)}{6}\right)$

$= \underline{\underline{\frac{1}{6}}}.$

4\. (a) $\lim_{x \to \frac{\pi}{2}} \left(\frac{\sin(x) - 1}{\cos(x)}\right)$ $\left[\frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to \frac{\pi}{2}} \left(\frac{\cos(x)}{-\sin(x)}\right)$

$= \frac{0}{-1} = \underline{\underline{0}}.$

(b) $\lim_{x \to 0^+} x \ln\left(1 + \frac{1}{x}\right)$

$= \lim_{x \to 0^+} \left(\frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}}\right)$ $\left[= \frac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0^+} \left(\frac{\frac{1}{1 + \frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}\right)$

$= \lim_{x \to 0^+} \left(\frac{1}{1 + \frac{1}{x}}\right) = \frac{1}{\infty} = \underline{\underline{0}}.$

(c) $\lim_{x \to 1} \dfrac{l(x) - (x-1)}{x-1}$ $\quad \left[= \dfrac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{1} = \dfrac{1-1}{1} = \underline{\underline{0}}.$

5(a) $\lim_{x \to \frac{\pi}{2}} \left(\tan(x) + \sec(x) \right) = \lim_{x \to \frac{\pi}{2}} \left(\dfrac{\sin(x)}{\cos(x)} + \dfrac{1}{\cos(x)} \right)$

$= \lim_{x \to \frac{\pi}{2}} \left(\dfrac{\sin(x) + 1}{\cos(x)} \right)$

$= \dfrac{1+1}{0} = \underline{\underline{\infty}}.$

The limit does not exist.

(b) $\lim_{x \to 1} \left(\dfrac{1}{\ln(x)} - \dfrac{1}{x-1} \right)$

$= \lim_{x \to 1} \left(\dfrac{x-1-\ln(x)}{(x-1)\ln(x)} \right)$ $\quad \left[= \dfrac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x \to 1} \left(\dfrac{1 - \frac{1}{x}}{\ln(x) + \frac{x-1}{x}} \right)$

$= \lim_{x \to 1} \left(\dfrac{x-1}{x\ln(x) + x - 1} \right)$ $\quad \left[= \dfrac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x \to 1} \left(\dfrac{1}{\ln(x) + \frac{x}{x} + 1} \right)$

$= \lim_{x \to 1} \left(\dfrac{1}{\ln(x) + 2} \right)$

$= \underline{\underline{\dfrac{1}{2}}}.$

(c) $\lim_{x\to 1}\left(\frac{\ln(x)}{x^2-x}\right)$ $\left[=\frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$=\lim_{x\to 1}\left(\frac{1/x}{2x-1}\right)$

$=\lim_{x\to 1}\left(\frac{1}{2x^2-x}\right) = \underline{\underline{1}}.$

6. L'Hospital's rule has been applied in the first step, viz. $\lim_{x\to 0}\frac{\cos(x)}{x^2}=\lim_{x\to 0}\frac{-\sin(x)}{2x}$, incorrectly. $\lim_{x\to 0}\frac{\cos(x)}{x^2} = \frac{1}{0}\,(=\infty)$, which is not of the form $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, i.e. an indeterminate form.

7. (a) $\lim_{x\to\infty}\left(\frac{1}{x}e^x\right)$ $\left[=\frac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$=\lim_{x\to\infty}\left(\frac{e^x}{1}\right) = \underline{\underline{\infty}}.$ The limit is undefined.

(b) $\lim_{x\to\infty}\frac{x^2}{e^x}$ $\left[=\frac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$=\lim_{x\to\infty}\frac{2x}{e^x}$ $\left[=\frac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$=\lim_{x\to\infty}\frac{2}{e^x} = \frac{2}{\infty} = \underline{\underline{0}}.$

(c) $\lim_{x\to 0^+}\sin(x)\ln(x)$. This is of the form $0\times\infty$ so we must manipulate.

$$\lim_{x\to 0^+}\left[\sin(x)\ln(x)\right]$$

$$= \lim_{x\to 0^+} \frac{\ln(x)}{1/\sin(x)} \qquad \left[= \frac{-\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0^+} \frac{1/x}{-\frac{\cos(x)}{\sin(x)}}$$

$$= \lim_{x\to 0^+}\left(-\frac{\sin^2(x)}{x\cos(x)}\right) \qquad \left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0^+}\left(-\frac{2\sin(x)\cos(x)}{\cos(x)-x\sin(x)}\right) = \frac{0}{1} = \underline{\underline{0.}}$$

8. (a) $$\lim_{x\to 0} \frac{x-\sin(x)}{x^2e^x} \qquad \left[= \frac{0-0}{0\times 1} = \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0} \frac{1-\cos(x)}{2xe^x+x^2e^x} \qquad \left[= \frac{1-1}{0+0} = \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0} \frac{-\sin(x)}{2e^x+2xe^x+2xe^x+x^2e^x}$$

$$= \lim_{x\to 0} \frac{-\sin(x)}{e^x(2+4x+x^2)} = \frac{0}{1(2+0+0)} = \underline{\underline{0.}}$$

(b) $\lim_{x\to 0} \frac{1-\cos(x)}{\sin^2(x)}$ $\left[= \frac{1-1}{0} = \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \frac{\sin(x)}{2\sin(x)\cos(x)}$

$= \lim_{x\to 0} \frac{1}{2\cos(x)} = \underline{\underline{\frac{1}{2}}}.$

(c) $\lim_{x\to 1} \left(\frac{x^4-7x^3+8x^2-2}{x^3+5x-6}\right).$

$= \lim_{x\to 1} \left(\frac{(x-1)(x^3-6x^2+2x+2)}{(x-1)(x+6)}\right)$

$= \lim_{x\to 1} \left(\frac{x^3-6x^2+2x+2}{x+6}\right) = \frac{1-6+2+2}{1+6} = \underline{\underline{-\frac{1}{7}}}.$

[Note: $\lim_{x\to 1} \left(\frac{x^4-7x^3+8x^2-2}{x^3+5x-6}\right) = \frac{0}{0}$, so we can, alternatively, use L'Hospital's rule.]

(d) $\lim_{x\to 0} \left(\operatorname{cosec}(x) - \frac{1}{x}\right)$

$= \lim_{x\to 0} \left(\frac{x-\sin(x)}{x\sin(x)}\right)$ $\left[= \frac{0-0}{0} = \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \left(\frac{1-\cos(x)}{\sin(x)+x\cos(x)}\right) = \left[\frac{1-1}{0+0} = \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \left(\frac{\sin(x)}{2\cos(x)-x\sin(x)}\right)$

$= \frac{0}{2-0} = \underline{\underline{0}}.$

(e) $\lim_{x \to 0} x^2 \ln(x) = 0 \times -\infty$ so we must manipulate.

$= \lim_{x \to 0} \frac{\ln(x)}{1/x^2}$ $\left[= \frac{-\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \frac{1/x}{-2/x^3} = \lim_{x \to 0} \left(-\frac{1}{x} \times \frac{x^3}{2}\right) = \lim_{x \to 0} \left(-\frac{x^2}{2}\right) = \underline{\underline{0.}}$

(f) $\lim_{x \to 0} \left(\frac{e^x + e^{-x} - x^2 - 2}{\sin^2(x) - x^2}\right) = \frac{1 - 1 - 0 - 2}{0 - 0} = \infty.$

The limit does not exist.

(g) $\lim_{x \to 0} \frac{\cot(x)}{\cot(2x)} = \lim_{x \to 0} \left(\frac{\cos(x)}{\sin(x)} \times \frac{\sin(2x)}{\cos(2x)}\right)$

$= \lim_{x \to 0} \left(\frac{2\cos^2(x)}{\cos(2x)}\right) = \underline{\underline{2.}}$

(h) $\lim_{x \to \infty} \left(\frac{5x + 2\ln(x)}{x + 3\ln(x)}\right)$ $\left[= \frac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to \infty} \left(\frac{5 + 2/x}{1 + 3/x}\right) = \underline{\underline{5.}}$

(i) $\lim_{x \to 0} \left(\frac{\cos(2x) - \cos(x)}{\sin^2(x)}\right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \left(\frac{-2\sin(2x) + \sin(x)}{2\sin(x)\cos(x)}\right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x \to 0} \left(\frac{-4\cos(2x) + \cos(x)}{2\cos(2x)}\right)$ (since $2\sin(x)\cos(x) = \sin(2x)$)

$= \frac{-4 + 1}{2} = \underline{\underline{-3/2}}.$

9. (a) i. $\lim_{x \to 8} \left(\frac{x-8}{\sqrt[3]{x}-2} \right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x \to 8} \left(\frac{1}{\frac{1}{3}x^{-2/3} - 2} \right)$

$= \lim_{x \to 8} \left(\frac{1}{\frac{1}{3x^{2/3}} - 2} \right)$

$= \lim_{x \to 8} \left(\frac{3\sqrt[3]{x^2}}{1 - 6\sqrt[3]{x^2}} \right) = \frac{3 \times 4}{1 - 6 \times 4} = -\frac{12}{23}.$

ii. $\lim_{x \to 1} \left(\frac{e^x - e}{x-1} \right)$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x \to 1} \left(\frac{e^x}{1} \right) = e.$

(b) Given $f(\pi) = 0$ and $\lim_{x \to \pi} \frac{f(x)}{\sin(x)} = 2$

Now $\lim_{x \to \pi} \frac{f(x)}{\sin(x)} = \frac{f(\pi)}{\sin(\pi)} = \frac{0}{0}$ which is indeterminate so we use L'Hospital's rule.

$\lim_{x \to \pi} \frac{f(x)}{\sin(x)} = \lim_{x \to \pi} \frac{f'(x)}{\cos(x)} = \frac{f'(\pi)}{-1} = -f'(\pi).$

But $\lim_{x \to \pi} \frac{f(x)}{\sin(x)} = 2$, so $f'(\pi) = -2$.

10. $$\lim_{x\to 0} x^{\sin(x)} = \lim_{x\to 0} e^{\sin(x)\ln(x)}$$

$$= \exp\left(\lim_{x\to 0} \frac{\ln(x)}{\frac{1}{\sin(x)}}\right)$$

Now $$\lim_{x\to 0} \frac{\ln(x)}{1/\sin(x)} \quad \left[= \frac{-\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0} \frac{1/x}{-\frac{\cos(x)}{\sin^2(x)}}$$

$$= \lim_{x\to 0} \left(-\frac{\sin^2(x)}{x\cos(x)}\right) \quad \left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0} \left(-\frac{2\sin(x)\cos(x)}{\cos(x) - x\sin(x)}\right) = 0.$$

$$\therefore \lim_{x\to 0} x^{\sin(x)} = e^0 = \underline{\underline{1.}}$$

11. $$(1+x)^{1/x} = e^{\ln(1+x)^{1/x}} = e^{\frac{1}{x}\ln(1+x)}$$

Now $$\lim_{x\to 0^+} \left(\frac{1}{x}\ln(1+x)\right) \quad \left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$$

$$= \lim_{x\to 0^+} \left(\frac{1}{1+x}\right) = 1$$

$$\therefore \lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = e^1 = \underline{\underline{e.}}$$

12. $\lim\limits_{x \to \infty} x\left(a^{1/x} - 1\right)$

$$= \lim_{x \to \infty} \frac{a^{1/x} - 1}{1/x} \qquad \left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$$

Now $\dfrac{d}{dx}\left(a^{1/x}\right) = \dfrac{d}{dx}\left(e^{\frac{1}{x}\ln(a)}\right)$

$$= e^{\frac{1}{x}\ln(a)}\left(-\frac{1}{x^2}\ln(a)\right)$$

$$= -a^{1/x} \cdot \frac{\ln(a)}{x^2}$$

So $\lim\limits_{x \to \infty} \dfrac{a^{1/x} - 1}{1/x} = \lim\limits_{x \to \infty} \dfrac{-a^{1/x}\ln(a)}{x^2} \cdot \dfrac{1}{-1/x^2}$

$$= \lim_{x \to \infty} a^{1/x}\ln(a) = 1 . \ln(a) = \underline{\underline{1.}}$$

13. (a) $\lim\limits_{x \to 1^+} x^{\frac{1}{x-1}}$

$$= \lim_{x \to 1^+} e^{\frac{1}{x-1}\ln(x)} = e^{\lim\limits_{x \to 1^+} \frac{\ln(x)}{x-1}}$$

Now $\lim\limits_{x \to 1^+} \dfrac{\ln(x)}{x} \qquad \left[= \dfrac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$$= \lim_{x \to 1^+} \frac{1}{x} = 1.$$

$\therefore \lim\limits_{x \to 1^+} x^{\frac{1}{x+1}} = e^1 = \underline{\underline{e.}}$

(b) $\lim_{x\to 0} (\sin(x))^x = \lim_{x\to 0} e^{x\ln(\sin(x))} = e^{\lim_{x\to 0} x\ln(\sin(x))}$

Now $\lim_{x\to 0} x\ln(\sin(x))$ (of the form $0\times\infty$)

$= \lim_{x\to 0} \dfrac{\ln(\sin(x))}{1/x}$ $\left[= \dfrac{-\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \dfrac{\cos(x)/\sin(x)}{-1/x^2}$

$= \lim_{x\to 0} \left(-\dfrac{x^2\cos(x)}{\sin(x)}\right)$ $\left[= \dfrac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \left(-\dfrac{2x\cos(x) - x^2\sin(x)}{\cos(x)}\right) = 0.$

So $\lim_{x\to 0} (\sin(x))^x = e^0 = \underline{\underline{1.}}$

(c) $\lim_{x\to \infty} (x+1)^{2/x} = \lim_{x\to \infty} e^{\frac{2}{x}\ln(x+1)} = e^{\lim_{x\to\infty} \frac{2}{x}\ln(x+1)}$

Now $\lim_{x\to \infty} \dfrac{2\ln(x+1)}{x}$ $\left[= \dfrac{\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to \infty} \dfrac{2}{x+1} = 0$

$\therefore \lim_{x\to \infty} (x+1)^{2/x} = e^0 = \underline{\underline{1.}}$

14. (a) $\lim\limits_{x\to 0}\left(\cos(x)\right)^{1/x} = \lim\limits_{x\to 0} e^{\frac{1}{x}\ln(\cos(x))}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\ln(\cos(x))}$

Now $\lim\limits_{x\to 0}\dfrac{\ln(\cos(x))}{x}$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim\limits_{x\to 0}\dfrac{-\frac{\sin(x)}{\cos(x)}}{1} = \lim\limits_{x\to 0}\left(-\tan(x)\right) = 0.$

So $\lim\limits_{x\to 0}\left(\cos(x)\right)^{1/x} = e^{0} = \underline{\underline{1.}}$

(b) $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{x} = \lim\limits_{x\to 0} e^{-x\log x}$

$= e^{\lim\limits_{x\to 0}(-x\log x)}$

Now, $\lim\limits_{x\to \infty}(-x\log x)$ $(0\times\infty)$

$= -\lim\limits_{x\to 0}\left(\dfrac{\log(x)}{1/x}\right)$ $\left[= \frac{-\infty}{\infty} \Rightarrow \text{L'Hospital's rule}\right]$

$= -\lim\limits_{x\to 0}\left(\dfrac{1/x}{-1/x^2}\right)$

$= \lim\limits_{x\to 0}(x) = 0.$

$\therefore \lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{x} = e^{0} = \underline{\underline{1.}}$

15.(a) $f(1)=1$ and $f'(1)=e$.

As $x \to 1$, $\dfrac{[f(x)]^4-1}{x^2-1} \to \dfrac{1-1}{1-1} = \dfrac{0}{0} \Rightarrow$ L'Hospital's rule.

$$\lim_{x\to1}\frac{[f(x)]^4-1}{x^2-1} = \lim_{x\to1}\frac{4[f(x)]^3 \cdot f'(x)}{2x}$$

$$= \frac{4\times[1]^3\times e}{2} = \underline{\underline{2e}}.$$

(b) $\displaystyle\lim_{x\to0}\frac{f(x)}{e^x-1} = e$ and $f(0)=0$.

As $x \to 0$, $\dfrac{f(x)}{e^x-1} \to \dfrac{f(0)}{e^0-1} = \dfrac{0}{0} \Rightarrow$ L'Hospital's rule.

$$\lim_{x\to0}\frac{f(x)}{e^x-1} = \lim_{x\to0}\frac{f'(x)}{e^x} = \frac{f'(0)}{e^0} = f'(0).$$

But $\displaystyle\lim_{x\to0}\frac{f(x)}{e^x-1} = e$, $\Rightarrow \underline{\underline{f'(0)=e}}.$

16. (a) $\lim_{x\to 0} \frac{\sin(\alpha x)}{\sin(\beta x)}$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule} \right]$

$= \lim_{x\to 0} \frac{\alpha \cos(\alpha x)}{\beta \cos(\beta x)} = \underline{\underline{\alpha/\beta}}.$

(b) $\lim_{x\to 0} \frac{\sin(x^\alpha)}{(\sin(x))^\alpha}$, $\alpha \in \mathbb{Z}^+$. $\beta \in \mathbb{Z}^+$.

The limit $= \frac{0}{0}$ so use L'Hospital's rule.

$\lim_{x\to 0} \frac{\sin(x^\alpha)}{(\sin(x))^\alpha}$

$= \lim_{x\to 0} \frac{\alpha x^{\alpha-1} \cos(x^\alpha)}{\alpha (\sin(x))^{\alpha-1} \cos(x)}$

$= \lim_{x\to 0} \left(\frac{x}{\sin(x)}\right)^{\alpha-1} \cdot \frac{\cos(x^\alpha)}{\cos(x)}$

$= (1)^{\alpha-1} \cdot \frac{1}{1}$

$= \underline{\underline{1}}.$

17. (a) $\lim_{x\to 0} \dfrac{\cos(\alpha x) - \cos(\beta x)}{x^2}$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \dfrac{-\alpha \sin(\alpha x) + \beta \sin(\beta x)}{2x}$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to 0} \dfrac{-\alpha^2 \cos(\alpha x) + \beta^2 \cos(\beta x)}{2} = \dfrac{1}{2}\left(\beta^2 - \alpha^2\right).$

(b) $\lim_{x\to \beta} \dfrac{\sin^2(x) - \sin^2(\beta)}{x^2 - \beta^2}$ $\left[= \frac{0}{0} \Rightarrow \text{L'Hospital's rule}\right]$

$= \lim_{x\to \beta} \dfrac{2\sin(x)\cos(x)}{2x} = \dfrac{1}{2\beta}\sin(2\beta).$

Exercise E.8.1

7. Determine the following limits, if they exist.

c $\lim_{x \to 0^+} (\cos x)(\ln x)$

8. Evaluate the following limits, if they exist.

(d) $\lim_{x \to 0}\left(\operatorname{cosec} x - \frac{1}{x}\right)$ (e) $\lim_{x \to 0} x^2 \ln x$ (f) $\lim_{x \to 0}\left(\frac{e^x - e^{-x} - x^2 - 2}{\sin^2 x - x^2}\right)$

(g) $\lim_{x \to 0}\left(\frac{\cot x}{\cot 2x}\right)$ (h) $\lim_{x \to \infty}\left(\frac{5x + 2\ln x}{x + 3\ln x}\right)$ (i) $\lim_{x \to 0}\left(\frac{\cos 2x - \cos x}{\sin^2 x}\right)$

9. (a) Determine i. $\lim_{x \to 8}\left(\frac{x - 8}{\sqrt[3]{x} - 2}\right)$ ii. $\lim_{x \to 1}\left(\frac{e^x - e}{x - 1}\right)$

(b) Consider the continuous function f with a continuous first derivative such that $f(\pi) = 0$.

Given that $\lim_{x \to \pi}\frac{f(x)}{\sin x} = 2$, calculate the value $f'(\pi)$.

10. Determine $\lim_{x \to 0} x^{\sin x}$. [Hint: Let $z = x^{\sin x}$ and take $\ln z$ to transform it to the form $\frac{\infty}{\infty}$].

11. Show that $(1 + x)^{1/x} = e^{\left(\frac{1}{x}\right)\ln(1 + x)}$. Hence, prove that $\lim_{x \to 0^+}(1 + x)^{1/x} = e$.

12. Determine $\lim_{x \to \infty} x(a^{1/x} - 1)$.

13. Determine the following limits.

(a) $\lim_{x \to 1^+} x^{\frac{1}{x-1}}$ (b) $\lim_{x \to 0}(\sin x)^x$ (c) $\lim_{x \to \infty}(x + 1)^{2/x}$

14. Determine (a) $\lim_{x \to 0}(\cos x)^{1/x}$ (b) $\lim_{x \to 0}\left(\frac{1}{x}\right)^x$.

15. (a) Given that $f(1) = 1$ and $f'(1) = e$, calculate $\lim_{x \to 1}\frac{[f(x)]^4 - 1}{x^2 - 1}$.

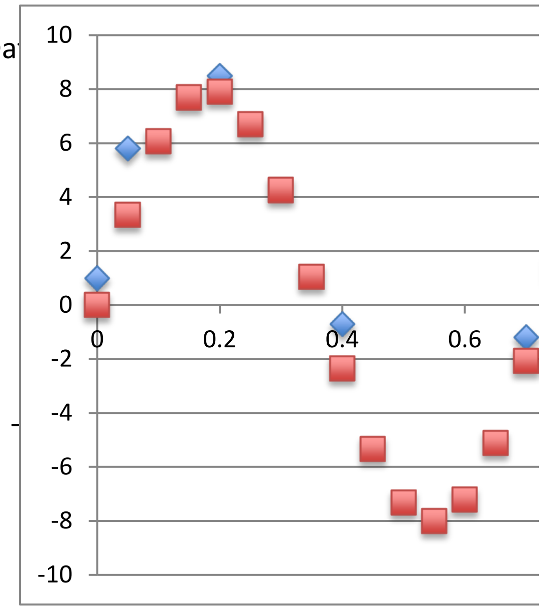
(b) Given that $\lim_{x \to 0}\frac{f(x)}{e^x - 1} = e$, where f is a continuous differentiable function, find $f'(0)$ given that $f(0) = 0$.

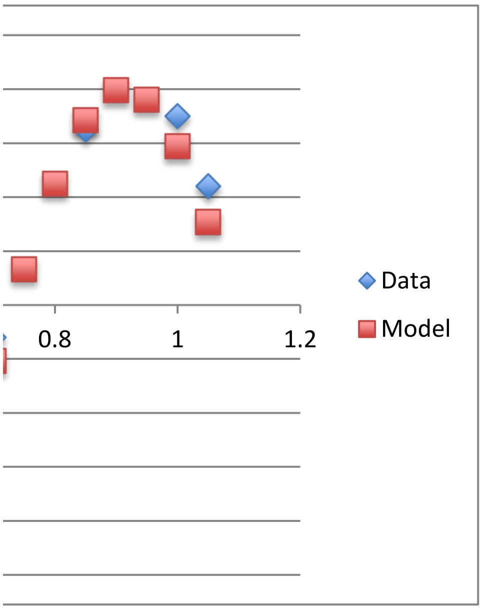
16. Evaluate (a) $\lim_{x \to 0}\frac{\sin \alpha x}{\sin \beta x}$ (b) $\lim_{x \to 0}\frac{\sin x^\alpha}{(\sin x)^\alpha}$, where $\alpha \in \mathbb{Z}^+$ and $\beta \in \mathbb{Z}^+$.

17. Evaluate (a) $\lim_{x \to 0}\frac{\cos \alpha x - \cos \beta x}{x^2}$ (b) $\lim_{x \to \beta}\frac{\sin^2 x - \sin^2 \beta}{x^2 - \beta^2}$

Damped Oscillation		A	8
		n	8.6
	13.15	c	0

Time	Time elap	Displacement		0.05		
				Time	Da	
13.15	0	1		0		
13.21	0.06	5.8		0.05		
13.34	0.19	8.5		0.1		
13.54	0.39	-0.7		0.15		
13.83	0.68	-1.2		0.2		
13.95	0.8	3.1		0.25		
14.02	0.87	6.5		0.3		
14.14	0.99	7		0.35		
14.2	1.05	4.4		0.4	-	
14.36	1.21	-3.9		0.45		
14.73	1.58	4.2		0.5		
15.35	2.2	-4.5		0.55		
16.83	3.68	-5.3		0.6		
18.04	4.89	5.8		0.65		-5.112
19.13	5.98	-5.7		0.7	-1.2	-2.081
				0.75		1.3283
				0.8		4.4961
				0.85	6.5	6.8452
				0.9		7.9481
				0.95		7.6038
				1	7	5.8752
				1.05	4.4	3.0768




0.8
1
1.2
Data
Model

Core Errata

Core Supplement